Nuclear Accelerators
Overview

Electrostatic Accelerators

Cascade generator
→ steady (DC) beam, high quality focusing, energy, currents; but low energies

Van de Graaff V.d.G. Tandem Accelerator 2-3 stages

Electrodynamic Accelerators

Cyclotrons conventional, sector-focusing
→ pulsed (AC) beam, high energies but lower quality focusing, energy definition, lower currents

Synchrotons Wideröe, Alvarez Linacs

Advanced Technology Accelerators

New principles: Collective acceleration, wake-field acceleration conceptual stage
Cockcroft-Walton (Cascade) Generator

Voltage multiplier
(Greinacher, 1921; Cockcroft-Walton, 1932.)

\[ \Delta U = \frac{Q}{C} \]

\[ U_0 \sin \omega t \]

Transformer

Ground

Rectifier

\[ B_1 \]

\[ A_1 \]

\[ A_0 \]

\[ U_{B1} \]

\[ U_{A1} \]

\[ U_{A0} \]

\[ 2U_0 \]

\[ U_0 \]
**Cockroft-Walton Pre-Accelerator**

Voltage doubling for each stage. Typical range order ~ $n \cdot 100$ kV ~ $10$ MV

*CW* used here to charge an enclosed metal terminal containing the ion source. Provides pre-acceleration of ions produced in the source → allows magnetic analysis.
Van de Graaff Electrostatic Accelerators

Van de Graaff, 1929

Operating limitations: 2 MV terminal voltage in air, 18-20 MV in pressure tank with insulating gas (SF$_6$ or gas mixture N$_2$, CO$_2$)

Acceleration tube has equipotential plates connected by resistor chain (R), ramping field down.

Typical for a CN: 7-8 MV terminal voltage
Early Van de Graaf Accelerators

Fig. 15. The 2 m DTM Van de Graaff generator installed in the new DTM Laboratory. This photograph was taken on November 13, 1933. The drive belt as shown passes completely through the terminal from drive motors on the laboratory walls. The spray supply for the charging system is shown attached to the wall at the left. The manually operated shorting rod with a spherical termination is lying on the floor under this power supply. The multielectrode acceleration tube leads into the experimental research room below and the fishline controls for the ion source instrumentation lead downward slightly to the left of the acceleration tube. In the background observing the instrument are from the left M. A. Tuve, L. R. Hafstad, and O. Dahl.

Fig. 27. The column of one of the early HVEC Model CN Van de Graaff accelerators during initial assembly at the HVEC Burlington, Massachusetts factory.
Tandem Van de Graaffs

Built in the late 1950s, 2 stage and 3-stage (Brookhaven), to increase beam energy

Accelerator structure enclosed in pressurized tank (SF$_6$)  
Corona rings to homogenize electric field.
The Yale MP Tandem
Munich University MP Tandem

Ion Source  MP Tandem

Vacuum Beam Line

90° Deflection Magnet
Stripping Probability for Heavy Ions

Charge distribution

\[ f(q) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(q - \bar{q})^2}{2\sigma^2} \right\} \]

Average \( \bar{q} \), variance \( \sigma^2 \)

\[
\frac{\bar{q}}{Z} = \begin{cases} 
  avZ^{-1/2} & \text{for } \frac{\bar{q}}{Z} \leq 0.3 \\
  a = 0.16(He), \\
  0.18(N_2, Ar) \\
  \sigma = (0.3 - 0.4)Z^{0.4} \\
  \ln\left(\frac{v}{mZ^\alpha}\right) / \ln\left(nZ^\beta\right) & \text{for } 0.3 < \frac{\bar{q}}{Z} \leq 0.9 \\
  v = 1.38\sqrt{E/AMeV} & \text{velocity} 
\end{cases}
\]

Tabulation: Chaki&Elmore UR-NSRL-240, 1980
Linear Accelerators

Wideröe 1928, Alvarez 1946

Linear trajectory, no deflection magnet, no radiative losses

Hollow drift tubes, E-field-free interior, contain magn. focusing elements.

Accelerating gap between 2 drift tubes on different el. potential

\[ U(t) = U_0 \cdot \sin (\omega t) \quad 10\text{MHz-3GHz} \]
on all gaps \( \rightarrow \) alternating E field switch polarity while particle is hidden.

\[ L = v \cdot \frac{T}{2} = \beta \cdot \frac{\lambda}{2} \]

\[ \beta = \frac{v}{c} \]

After gap \( n \):

\[ v_n = \sqrt{n \frac{2(qU_0)}{m}} \quad \text{velocity} \]
**Acceleration with Linac**

Alternating directions of the gap fields, change while particle is hidden inside drift tube.

Q: Phase conditions change during acceleration, particle speed. Is continuous operation possible, without loss of particles?
Voltage encountered at gaps: \( t = \phi / \omega \)

Phase angle of particle: \( \phi = \phi_s + \omega (t - t_s) \), force \( F = qU_0 / (\text{gap length } L) \)

Stability analysis for \( \tilde{\phi} \approx 0 \)
Stable oscillations about synchr. phase for \( \cos \phi_s > 0 \),
\( \rightarrow \phi_s < 90^\circ \) inject during first quarter cycle!

\[
\frac{d^2}{dt^2} (\phi - \phi_s) + \frac{\omega F \cos \phi_s}{p_s} (\phi - \phi_s) = 0
\]

\[
\frac{d^2 \tilde{\phi}}{dt^2} + \omega^2 \phi = 0 \quad \text{differential equation}
\]

W. Udo Schröder, 2004
CERN Proton Linac
Cyclotrons
Charged particles in electromagnetic fields, $E, B$

**Lorentz Force**

$$F = q \cdot \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

$E = 0 : \quad F = \dot{p} = q \cdot \vec{v} \times \vec{B}$

$$\vec{p} = q \cdot \vec{r} \times \vec{B} \quad \text{orbit radius } r, \left( \vec{r} \perp \vec{B} \right)$$

$$p = q \cdot r \cdot B, \quad \text{equilibrium orbit at } r = \frac{p}{qB}$$

$$\vec{p} = m\vec{v} \quad \rightarrow \quad \vec{v} = \vec{\omega}_0 \times \vec{r}$$

$$\vec{\omega}_0 = -\frac{q}{m} \vec{B} \quad \text{Particle Cyclotron Frequency}$$
**Cyclotron Max. Energy**

**Cyclotron Frequency**

\[ \vec{\omega}_0 = -\frac{q}{m} \vec{B} \quad \text{same for all } v \]

Acceleration, if \( \omega_{\text{field}} = \omega_0 \)

Equilibrium orbit \( r: \quad p = qBr \)

\( \rightarrow \) maximum \( p_{\text{max}} = qBR \)

**Maximum Energy**

\[ \varepsilon_{\text{max}} = \frac{(qBR)^2}{2m} = K \cdot \frac{q^2}{A} \]

**Relativistic effects:** \( m \Rightarrow W = \varepsilon + m_0c^2 \) shape B field to compensate. Defocusing corrected with sectors and fringe field.
Cyclotron Resonance Condition

$\leftarrow f=1$, matched

Accelerating rf voltage has to be switched with frequency of gap crossings:

$$\omega_{rf} = \omega_{0, \text{particle}} = \frac{qB}{m}$$

$$f = \frac{\omega_{rf}}{\omega_{0, \text{particle}}} = 1!$$

$\leftarrow f=1.1$, mismatched
Relativistic Correction

Particle Cyclotron Frequency

\[ \omega_0 = \frac{q}{m} B \text{ for small velocities } v \]

Relativistically: \( W = mc^2 = m_0 c^2 + \varepsilon_{\text{kin}} \)

\[ \omega_0 \rightarrow \omega(W) = \frac{qc^2}{W} B \text{ for relat. velocities } v \]

\[
\frac{d}{dW} \omega(W) = -\frac{\omega(W)}{W}
\]

decreases with incr. \( \varepsilon_{\text{kin}} \) \( \frac{\Delta W}{W} = -\frac{\Delta \omega}{\omega} \)

Example: projectile \( m_0 c^2 = A \cdot 938 \text{ MeV} \), energy change \( \Delta \varepsilon_{\text{kin}} = 20 \cdot A \text{ MeV} \)

\[ \Delta \omega/\omega = -\Delta W/W = 20 \text{ MeV}/938 \text{ MeV} = 0.02 \sim{\Delta \phi = -7.7}^0 \text{ per turn} \]

out of phase within 12 turns! Correction: increase \( B(r) \) radially.

Undesirable consequence: defocusing of beam in inhomogeneous \( B \)!
Focusing in Inhomogeneous Fields

\[ F_z = \dot{p}_z = q \cdot \left( \vec{v} \times \vec{B} \right)_z = qvB_r \]

\[ F_r = \dot{p}_r - mr \dot{\theta}^2 = q \cdot \left( \vec{v} \times \vec{B} \right)_r = -qvB_z \]

Particle prescribes “Betatron Oscillations”

\[ m \ddot{\zeta} + n \omega_0 (qB_0) \zeta = 0 \]

\[ \zeta = z / r_0 \quad \omega_r = \sqrt{1 - n \omega_0} \]

\[ m \ddot{\rho} + m \omega_0^2 [1 - n] \rho = 0 \]

\[ \rho = r / r_0 \quad \omega_z = n^{1/2} \omega_0 \quad \omega_0 = v / r_0 \]

Does not correct for relativistic increase in m!
The Sector-Focused Cyclotron

How to make B both radially concave (to correct for relativistic m) and convex (to have the desired fringe field bulge)?

In transition regions between magnetic peaks and valleys, $v_r \neq 0$. Particle drifts radially

Exit:
Above median plane $z > 0$: $F_z = -qv_r B_\theta < 0$ focus
below plane, $z < 0$ $F_z = -qv_r B_\theta > 0$ focus

Entrance: defocus

Adjust directions of $B_r$ and $B_\theta$ by shaping the pole faces: spiral arms
From Ion Source to Cyclotron

Movie from: NSCL/MSU
The End