NUCLEAR SPECTROSCOPY SURVEY
Chemical Evolution of our Universe

- Afterglow Light Pattern 380,000 yrs.
- Dark Ages
- Development of Galaxies, Planets, etc.
- Inflation
- Quantum Fluctuations

- 1st Stars about 400 million yrs.

Big Bang Expansion

13.7 billion years

W. Udo Schröder, NCSS 2012
Chemical Evolution of our Universe

Afterglow Light Pattern 380,000 yrs.

Dark Ages

Development of Galaxies, Planets, etc.

Inflation

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Probes for Nuclear Structure

To “see” an object, the wavelength $\lambda$ of the light used must be shorter than the dimensions $d$ of the object ($\lambda \leq d$).

(DeBroglie: $p = \hbar k = \hbar 2\pi/\lambda$)

Rutherford’s scattering experiments

$d_{\text{Nucleus}} \sim \text{few } 10^{-15} \text{ m}$

Need light of wave length $\lambda < 1 \text{ fm}$, equivalent energy $E$

\[ \text{Photons: } E = pc = \hbar c = 2\pi \frac{\hbar c}{\lambda} \geq 6 \cdot \frac{200 \text{MeV} \cdot \text{fm}}{1 \text{fm}} = 1.2 \text{ GeV} \]

Massive ($m \approx Am_p$) particle, e.g., proton ($A = 1$), $m_p c^2 \approx 0.9 \text{GeV}$:

\[ E = \frac{p^2}{2m} = \frac{(\hbar k)^2}{2m} \approx \frac{(\hbar c k)^2}{2Am_p c^2} \geq \frac{(200 \text{MeV} \cdot \text{fm})^2}{1.8 \cdot A \text{GeV}} \cdot \frac{(2\pi)^2}{\lambda^2} \]

\[ = \frac{80 \cdot 10^4 \text{MeV}^2 \text{fm}^2}{A \text{GeV}} \cdot \frac{1}{\text{fm}^2} = \frac{800}{A} \text{ MeV} \]

Heavy ions ($A \sim 100$): $\lambda \ll 1 \text{ fm} \rightarrow \text{classical particles}$

Scan energy states of nuclei. Bound systems have discrete energy states $\rightarrow$ unbound $E$ continuum

Not easily available as light

Can be made with charged particle accelerators
What Structure? - Spectroscopic Goals

Bound systems have discrete energy states $\rightarrow$ unbound $E$ continuum $\rightarrow$ Scan energy states of nuclei.

Desired information on nuclear states ("good quantum numbers"):

- Energy relative to ground state (g.s.)
  $(E_i, i=0,1,...; E^*)$
- Stability, life time against various decay modes ($\alpha, \beta, \gamma,...$)
- Electrostatic moments $Q_j \rightarrow$ see below
- Magnetostatic moments $M_i \rightarrow$ see below
- Angular momentum (nuclear spin)
- Parity (=spatial symmetry of quantum $\Psi$)
- Nucleonic (neutron & proton) configuration (e.g., "isospin" quantum #)
Particle and $\gamma$ Spectroscopy

Identify scattered/transmuted projectile & target nuclei, measure m, E, A, Z, I,.. of all ejectiles (particles and $\gamma$-rays).

Individual or multi detector setups, spectrometers
Off-line activation measurements
How to Excite Nuclei: Inelastic Nuclear Reactions

Coulomb excitation of rotational motion for deformed targets or vibrations.

\[ \text{Defl. angles } \theta, \phi \rightarrow \Delta \vec{L} \]

\( p, n, \ldots \) transfer reactions, e.g., \((d, n), (^3\text{He}, d)\).

Final state is excited

\[ \text{Defl. angles } \theta, \phi \rightarrow \Delta \vec{L} \]

Fusion/compound nucleus reactions,
Final state is highly excited, thermally metastable.
Coulomb Excitation (Semi-Classical)

\[ R_0 = \eta \lambda (1 + 1/\sin(\theta/2)) \quad \theta = 2 \cdot \arctan(\eta/L) \]

**Adiabaticity Condition:**
Fast “kick” will excite nucleus

\[ \text{initial } E_i^* \rightarrow \text{final } E_f^* \]

Otherwise adiabatic reorientation of deformed nucleus to minimize energy

**Collision time \( \ll \) rot. period**

\[ \tau_{\text{coll}} \approx \frac{2R_0}{v_{\text{rel}}} \ll \sigma_{\text{if}}^{-1} = \left[ \frac{1}{\hbar} (E_f^* - E_i^*) \right]^{-1} \]

**Adiabaticity parameter**

\[ \zeta := \frac{\eta \lambda}{v_{\text{rel}}} (E_f^* - E_i^*) = \frac{1}{4} \frac{\tau_{\text{coll}} \cdot \sigma_{\text{if}}}{1} \leq 1 \]

**Sommerfeld Parameter**

\[ \eta = \frac{e^2 Z_1 Z_2}{\hbar \nu_\omega} \]

Torque exerted on deformed target \( \Rightarrow \) excitation of collective nuclear rotations
Transfer of angular momentum from relative motion:

\[ I = 0 \rightarrow I = \Delta L = \hbar \Delta \ell \]

Angular dependence of excitation probability:

\( \Rightarrow \) for small energy losses
(weak excitations: \( E_i^* \rightarrow E_f^* \)

\[ \frac{d\sigma_{i\rightarrow f}(\theta)}{d\Omega} \approx P_{i\rightarrow f} \cdot \frac{d\sigma_{\text{Ruth}}(\theta)}{d\Omega} \]

Excitation probability in perturbation theory:

\[ P_{i\rightarrow f} \approx \sum_f \left| b_{i\rightarrow f} \right|^2 \quad b_{i\rightarrow f} = -i \int_0^\infty dt e^{i \hbar (E_f^* - E_i^*) t} \langle f | H'(t) | i \rangle \]

Transition amplitude \( \propto \) Fourier transform of transition ME
elm projectile - target interaction \( H' \leftrightarrow \rho \cdot V_{\text{Coul}} + \mathbb{J} \cdot \mathbb{A} \)
### Observation of Collective Nuclear Rotations

**Quadrupole moment (Deviation from spherical nucleus)**

\[
R(\theta, \phi) = R_0 \left[ 1 + \beta Y_0^2(\theta, \phi) \right]
\]

\[
\beta = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R} \quad \bar{R} = \frac{a + b}{2} ; \ \Delta R = a - b
\]

**Deexcitation-Gamma Spectra**

- **Measured energies (keV)**
  - E\(^{242}\text{Pu}\): 1084, 1114, 1143, 1167, 1231, 1268, 1318, 1359, 1431, 1467, 1506, 1545, 1585
  - E\(^{244}\text{Pu}\): 1133, 1201, 1242, 1284, 1326, 1368

**Typical values**

\[
\frac{\hbar^2}{2I} \approx 15 - 18 \text{keV}
\]

- **Rigid body moment of inertia**:
  - \( I_{\text{rig}} = \frac{2}{5} MR_0^2 \left[ 1 + 0.31 \beta \right] \)

- **Hydro-dynamical**:
  - \( I_{\text{irr}} = \frac{9}{8\pi} MR_0^2 \beta \)

- **Even I only**

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W. Udo Schröder, 2012

Wood et al., Heyde
Method: Particle Spectroscopy

Bound systems have discrete energy states $\rightarrow$ unbound $E$ continuum

Populate (excite) the spectrum of nuclear states.
Measure probabilities for excitation and de-excitation

Reconstruct energy states $\{E_i, I_i, \pi_i\}$ from energy, linear and angular momentum balance. Obtain structure information from probabilities.
Method: Simultaneous Absorption & Emission Spectroscopy

Reaction $^{64}\text{Ni}(p,p')^{64}\text{Ni}^*$: inelastic proton scattering, absorb $E$ from beam

$^{64}\text{Ni}_{gs} + \gamma_1 + \gamma_2 + \gamma_3 + ...$

Scattered Proton Spectrum $\rightarrow$ $p$-energy loss

Deexcitation $\gamma$ Spectrum

$\Delta E_p$
Example: Inelastic Particle Scattering

\[ ^{238}\text{U} + d \rightarrow ^{238}\text{U}^* + d' \]

Scattered deuteron kinetic energy spectrum

\[ E_{d}' = E_{d}' (\theta_d , E_d) \]

Coincident \( \gamma \) cascades indicate nuclear band structure (rotational, vibrational,...)
Measuring Energy Transfer: The Q Value Equation

How to interpret energies of scattered particles:

Lab System (target initially at rest, \( p = 0 \)):
\[
p_i = \sqrt{2m_iE_i} \quad \text{linear momentum}
\]
\[
p_3 \sin \theta_3 = p_4 \sin \theta_4 \quad y - \text{component}
\]
\[
p_3 \cos \theta_3 + p_4 \cos \theta_4 = p_1 \quad x - \text{component}
\]
\[
Q = E_3 + E_4 - E_1 = \text{Process Energy Release}
\]
e.g.: mass loss (\( Q > 0 \)), internal excitation (\( Q < 0 \))

Eliminate \( E_4, \theta_4 \) →

\[
Q(E_3, \theta_3) = E_3 \left(1 + \frac{m_3}{m_4}\right) - E_1 \left(1 - \frac{m_1}{m_4}\right) - \sqrt{E_3} \frac{2\sqrt{m_1m_3E_1}}{m_4} \cos \theta_3
\]

Measure kinetic energy \( E_3 \) vs. angle \( \theta_3 \) to determine reaction Q value. Predict energies of particles at different angles as functions of Q.
Transmutation in Compound Nucleus Reactions

Use mass tables to calculate $Q$

cm spectra of particles statistically emitted from CN (evaporated) are of Maxwell Boltzmann type

$$\frac{dN}{dE} \propto (E - E_B) \cdot e^{-E/T}$$

$E_B = \text{Coulomb barrier}$

$T = \text{effective nuclear temperature}$

Decay of CN populates states of the “evaporation residue” nucleus

Even for fixed $E^*$ the particles spectrum is continuous (Maxwell-Boltzmann), except for transitions to discrete spectrum at low $E_{ER^*}$
Simultaneous measurement of time of flight, specific energy loss and residual energy of particles
\( \rightarrow E, Z, A \)

Simultaneous measurement of specific energy loss \( \Delta E \) and residual energy \( E \) of particles
\( \rightarrow E, Z, (A) \)

Modified, after K.S. Krane, Introduction to Nuclear Physics, Wiley&Sons, 1988
Particle ID with Detector Telescopes

Particle ID (Z, A, E) Specific energy loss, spatial ionization density, TOF

Si Telescope Massive Reaction Products

\[ \Delta E - \Delta E \]

\[ E - \Delta E \]

SiSiCsI Telescope (Light Particles)

\[ \Delta E_{Si} \text{ Telescope} \]

\[ \Delta E_{Si} \text{ (ADC Channel)} \]

\[ E_{CsI} \text{ (ADC Channel)} \]

\[ \Delta E_{Si} (\text{ADC Channel}) \]

\[ E_{CsI} (\text{ADC Channel}) \]

\[ \theta_{lab} = 12^\circ \]

\[ ^{20}\text{Ne} + ^{12}\text{C} @ 20.5 \text{ MeV/u} \]

\[ ^{20}\text{Ne} + ^{12}\text{C} @ 20.5 \text{ MeV/u} - \theta_{lab} = 12^\circ \]
THE CHIMERA DETECTOR

Laboratori del Sud, Catania/Italy

### CHIMERA characteristic features

<table>
<thead>
<tr>
<th>Experimental Method</th>
<th>$\Delta E-E \rightarrow$ Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E-E$</td>
<td>E-TOF $\rightarrow$ Velocity, Mass</td>
</tr>
<tr>
<td>Pulse shape Method</td>
<td>$\rightarrow$ LCP</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Basic element</th>
<th>Si (300$\mu$m) + CsI(Tl) telescope</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Primary experimental observables</th>
<th>TOF $\delta t \leq 1$ ns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kinetic energy, velocity</td>
</tr>
<tr>
<td></td>
<td>$\delta E/E$ Light charged particles $\approx$2%</td>
</tr>
<tr>
<td></td>
<td>Heavy ions $\leq$ 1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total solid angle $\Delta \Omega/4\pi$</th>
<th>94%</th>
</tr>
</thead>
</table>

<table>
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<tr>
<th>Granularity</th>
<th>1192 modules</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Angular range</th>
<th>$1^\circ &lt; \theta &lt; 176^\circ$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Detection threshold</th>
<th>$&lt;0.5$ MeV/A for H.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\approx 1$ MeV/A for LCP</td>
</tr>
</tbody>
</table>

**Reversal Experimental Apparatus**

- **688 telescopes**
- **30°**
- **1°**
- **1m**

**Chimera mechanical structure**
Technology in $\gamma$ spectroscopy Greta HPGe Array

Realistic coverage: $\Omega/4\pi=0.8$
Spatial resolution: $\Delta x=2$ mm

GRETA Detector
Configuration
Two types of irregular hexagons, 60 each,
3 crystals/cryostat=40 modules
Detector - target distance = 15 cm
$750k$ → GRETINA
**$\gamma$-Ray Tracking with HPGe Detector Array**

Use Compton scattering laws to identify & track interactions of all $\gamma$'s. Conventional method requires many detectors to retain high resolution and avoid summing.

\[ E'_\gamma(\theta) = E_\gamma \cdot \frac{1}{1 + \left(\frac{E_\gamma}{m_e c^2}\right)[1 - \cos \theta]} \]

- **Compton Suppressed Ge**
  - $N_{\text{det}} = 100$
  - Peak efficiency = 0.1
  - Efficiency limited
- **Ge Sphere**
  - $N_{\text{det}} = 1000$ (summing)
  - Peak efficiency = 0.6
  - Too many detectors
- **Gamma Ray Tracking**
  - $N_{\text{det}} = 100$
  - Peak efficiency = 0.6
  - Segmentation

I.-Y. Lee, LBNL, 2011
Single-particle spectra: Irregular sequence, half-integer spins, various parities.

Collective vibrations: $\Omega^+ \text{g.s.}$, even spins & parity, regular sequence with bunching of $(0^+, 2^+, 4^+) \text{ triplet}$

Collective rotations: Regular quadratic sequence $0^+, 2^+, 4^+, \ldots$. Only even or only odd spins, $\Delta I=2$, uniform parity.
Different densities of states.

Similarities in energies, spin, parity sequence for mirror nuclei:
\((N, Z) = (a, b) = (b, a)_{\text{mirror}}\)
Gamma Decay of Isobaric Analog States

For same $T$, wfs for protons and neutrons are similar

→ “Mirror Nuclei” $^{19}\text{Ne}$ and $^{19}\text{F}$
Examples of Level Schemes: SM vs. Collective

- SM
- Collective

Level schemes for different elements:

- $^{16}_{6}$O
- $^{17}_{8}$O
- $^{18}_{8}$O
- $^{106}_{46}$Pd
- $^{242}_{92}$Pu

Energy levels (MeV) and spin labels:

- $0^+$, $1^-$, $2^+$, $3^-$, $3/2^+$, $5/2^-$, $1/2^+$
- $16^+$, $14^+$, $12^+$, $10^+$, $8^+$, $6^+$, $4^+$, $2^+$
Example of $\gamma$-particle Spectroscopy
Example

α-Decay of $^{241}$Am, subsequent γ emission from daughter.

Find coincidences $(E_\alpha, E_\gamma)$

9 γ-rays, 5 α
Example

$\alpha$-Decay of $^{241}$Am, subsequent $\gamma$ emission from daughter.

Find coincidences $(E_\alpha, E_\gamma)$

9 $\gamma$-rays, 5 $\alpha$
Example

$^{237}\text{Np}$ $^{241}\text{Am}$

5.378 MeV
5.433 MeV
5.476 MeV
5.503 MeV
5.546 MeV

γ Energy (keV)

α Energy (keV) - 5 MeV

W. Udo Schröder, 2012
No $\alpha-\gamma$ coincidences! $\rightarrow$ must be g.s. transition
Exercises: Nuclear Electrostatic Moments

Consider a nucleus with a homogeneous, axially symmetric charge distribution $\rho(\vec{r})$

1. Show that its electric dipole moment $Q_0$ is zero.

2. Assume for the charge distribution a homogenously charged rotational ellipsoid with semi axes $a$ and $b$, given by $(x^2 + y^2)/a^2 + z^2/b^2 = 1$.
   
   Show that 
   
   $$Q_0 = \frac{2}{5} eZ(b^2 - a^2) = \frac{4}{5} eZ\bar{R} \cdot \Delta R = \frac{4}{5} eZ\bar{R}^2 \cdot \delta$$

   $$\bar{R} = \frac{1}{2}(a + b); \quad \Delta R = b - a; \quad \delta = \Delta R/\bar{R}$$

3. From the measured $^{242}$Pu and $^{244}$Pu $\gamma$ spectra determine the deformation parameters $\beta$ of these two isotopes.
Alpha-Gamma Spectroscopy

- $^{251}\text{Fm} \rightarrow ^{247}\text{Cf}$

### Table 8.3 $\alpha$ Decays from $^{251}\text{Fm}$

<table>
<thead>
<tr>
<th>$\alpha$ Group</th>
<th>$\alpha$ Energy (keV)</th>
<th>Decay Energy (keV)</th>
<th>Excited-State Energy (keV)</th>
<th>$\alpha$ Intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>7305 ± 3</td>
<td>7423</td>
<td>0</td>
<td>1.5 ± 0.1</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>7251 ± 3</td>
<td>7368</td>
<td>55</td>
<td>0.93 ± 0.08</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>7184 ± 3</td>
<td>7300</td>
<td>123</td>
<td>0.29 ± 0.03</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>7106 ± 5</td>
<td>7221</td>
<td>202</td>
<td></td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>6928 ± 2</td>
<td>7040</td>
<td>383</td>
<td></td>
</tr>
<tr>
<td>$\alpha_6$</td>
<td>6885 ± 2</td>
<td>6996</td>
<td>427</td>
<td></td>
</tr>
<tr>
<td>$\alpha_7$</td>
<td>6833 ± 2</td>
<td>6944</td>
<td>479</td>
<td></td>
</tr>
<tr>
<td>$\alpha_8$</td>
<td>6782 ± 2</td>
<td>6892</td>
<td>531</td>
<td></td>
</tr>
<tr>
<td>$\alpha_9$</td>
<td>6762 ± 3</td>
<td>6872</td>
<td>552</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{10}$</td>
<td>6720 ± 3</td>
<td>6829</td>
<td>594</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>6681 ± 4</td>
<td>6789</td>
<td>634</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{12}$</td>
<td>6638 ± 3</td>
<td>6745</td>
<td>678</td>
<td>0.36 ± 0.06</td>
</tr>
<tr>
<td>$\alpha_{13}$</td>
<td>6579 ± 3</td>
<td>6686</td>
<td>738</td>
<td>0.26 ± 0.04</td>
</tr>
</tbody>
</table>

### Table 8.4 $\gamma$ Rays in $^{247}\text{Cf}$ following $\alpha$ Decay of $^{251}\text{Fm}$

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Intensity (% of decays)</th>
<th>Energy (keV)</th>
<th>Intensity (% of decays)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.0 ± 0.2</td>
<td>0.58 ± 0.08</td>
<td>425.4 ± 0.1</td>
<td>51 ± 4</td>
</tr>
<tr>
<td>67.1 ± 0.2</td>
<td>0.28 ± 0.05</td>
<td>477.0 ± 0.3</td>
<td>0.54 ± 0.08</td>
</tr>
<tr>
<td>122.1 ± 0.2</td>
<td>0.28 ± 0.05</td>
<td>480.4 ± 0.1</td>
<td>21 ± 2</td>
</tr>
<tr>
<td>331.0 ± 0.3</td>
<td>0.35 ± 0.07</td>
<td>496 ± 1</td>
<td>~ 0.08</td>
</tr>
<tr>
<td>358.3 ± 0.1</td>
<td>17 ± 1.5</td>
<td>616 ± 1</td>
<td>~ 0.05</td>
</tr>
<tr>
<td>372.2 ± 0.4</td>
<td>0.25 ± 0.05</td>
<td>623.0 ± 0.8</td>
<td>0.07 ± 0.02</td>
</tr>
<tr>
<td>382.2 ± 0.3</td>
<td>1.2 ± 0.13</td>
<td>678.0 ± 0.8</td>
<td>0.26 ± 0.06</td>
</tr>
<tr>
<td>410.0 ± 0.3</td>
<td>0.50 ± 0.07</td>
<td>683 ± 1</td>
<td>~ 0.04</td>
</tr>
</tbody>
</table>

**Coincidence Gate**

$\gamma$ Rays (keV):
- $\alpha_5$: 383.2
- $\alpha_6$: 372.2, 383.2
- $\alpha_7$: 55.0, 67.1, 122.1, 358.3, 425.4, 480.4
- $\alpha_8$: 331.0, 358.3, 410.0, 425.4, 477.0, 480.4
- $\alpha_{12}$: 623.0, 678.0
γ Spectroscopy
Theoretical Tools
Spectroscopic vs. Intrinsic Quadrupole Moment

I≠0: Transformation body-fixed to lab system

\[ Q_z = \left\langle m_I = I \left| \hat{Q} \right| m_I = I \right\rangle = \frac{1}{2} \left( 3 \cos^2 \theta - 1 \right) Q_0 \]

\[ m_I = I = \sqrt{I(I+1)} \cos \theta \quad \rightarrow \quad \cos \theta = \frac{I}{\sqrt{I(I+1)}} \]

\[ Q_z(m_I) = \frac{3m_I^2 - I(I+1)}{I(2I-1)} \cdot Q_z(m_I = I) \]

Any orientation

"The" quadrupole moment

\[ Q_z = \frac{2I-1}{2(I+1)} \cdot Q_0 \quad (= 0 \text{ for } I = 0, 1/2) \]
Electromagnetic Radiation

Protons in nuclei = moving charges $\rightarrow$ emits electromagnetic radiation, except if nucleus is in its ground state!

Propagating Electric Dipole Field

E. Segré: *Nuclei and Particles*, Benjamin&Cummins, 2nd ed. 1977
Nuclear Electromagnetic Transitions

Conserved: Total energy \( (E) \), total angular momentum \( (I) \) and total parity \( (\pi) \):

\[
E_i = E_f + E_\gamma \quad \vec{I}_i = \vec{I}_f + \vec{I}_\gamma \quad \pi_i = \pi_f \cdot \pi_\gamma
\]

**Initial Nucl. State**

\[ j_{22}(r) \cdot Y_{m_j}^2(\theta, \varphi) \]

**Final Nucl. State**

\[ j_{11}(r) \cdot Y_{m_f}^1(\theta, \varphi) \]

**Photon WF**

\[ \psi_{\Delta E}(r_\gamma) \cdot Y_{\mu=\Delta m}^{\ell=\Delta I}(\theta, \varphi) \]

Initial \( N_i \) spatial symmetry \( \rightarrow \) retained in overall combination \( N_f + \gamma \)

Consider often only *electric* multipole transitions.
Neglect weaker *magnetic* transitions due to changes in current distributions.
Selection Rules for Electromagnetic Transitions

Conserved: Total energy \((E)\), total angular momentum \((I, I_z)\), total parity \((\pi)\):

\[
E_i = E_f + E_\gamma \quad \vec{I}_i = \vec{I}_f + \vec{I}_\gamma \quad \pi_i = \pi_f \cdot \pi_\gamma
\]

Quantization axis: \(z\) direction. Physical alignment of \(I\) possible (B field, angular correlation)

Coupling of Angular Momenta

Angular momenta \(\vec{I}_i, \vec{I}_f\) direction undetermined.

Projections conserved \(m_i, m_f, \quad m_\gamma = m_i - m_f\)

\[
|\vec{I}_i - \vec{I}_f| \leq |\ell_\gamma| \leq |\vec{I}_i + \vec{I}_f|
\]

Electric dipole radiation \([\ell_\gamma = 1(\hbar)]:\)

\(I_f = I_i, \quad I_i \pm 1(\hbar) \quad m_f = m_i, m_i \pm 1\)

Other types (magnetic) and multipolarities have other selection rules.
Transition Probabilities: Weisskopf’s s.p. Estimates

Consider single nucleon in circular orbit (extreme SM)

Low – energy / long wave length limit: \(k_\gamma \cdot R_{\text{Nucleus}} \ll 1, \quad R_{\text{Nucleus}} \leq 10 \, \text{fm}\)

\[E_\gamma = (\hbar c) \cdot k_\gamma \approx (200 \, \text{MeV fm}) \cdot k_\gamma \ll \frac{200 \, \text{MeV fm}}{10 \, \text{fm}} = 20 \, \text{MeV}\]

\[j_\ell(kr) \approx \frac{(kr)^\ell}{(2\ell + 1)!!} \quad \text{for} \quad kr \ll 1 \rightarrow j_{\ell+1}(kr) \ll j_{\ell-1}(kr) \quad (k := k_\gamma)\]

\[
P(E_\ell) \approx \frac{4.4(\ell + 1)}{\ell \left[ (2\ell + 1)!! \right]^2} \left( \frac{3}{\ell + 3} \right)^2 \left( \frac{\hbar \omega}{197 \, \text{MeV}} \right)^{2\ell+1} \left( \frac{R}{\text{fm}} \right)^{2\ell} \frac{10^{21}}{\text{s}}
\]

\[
P(M_\ell) \approx \frac{1.9(\ell + 1)}{\ell \left[ (2\ell + 1)!! \right]^2} \left( \frac{3}{\ell + 2} \right)^2 \left( \frac{\hbar \omega}{197 \, \text{MeV}} \right)^{2\ell+1} \left( \frac{R}{\text{fm}} \right)^{2\ell-2} \frac{10^{21}}{\text{s}}
\]
Weisskopf’s $E\ell$ Estimates

**single particle**: 

$$P(E\ell) \sim E^{2\ell+1} \cdot A^{2/3}$$

**Experimental $E1$:**

Factor $10^3$-$10^7$ slower than s.p. WE →
Configurations more complicated than s.p. model, time required for rearrangement

**Experimental $E2$:**

Factor $10^2$ faster than s.p. WE →
Collective states, more than 1 nucleon.

**Experimental $E\ell$ ($\ell>2$):**

App. correctly predicted.

$T_{1/2}$ for solid lines have been corrected for internal conversion.
Weisskopf’s $M\ell$ Estimates

**single particle**:

$P(M\ell) \sim E_\gamma^{2\ell+1} \cdot A^{(\ell-1)2/3}$

**Experimental $M\ell$:**

Several orders of magnitude weaker than $E\ell$ transitions.

$T_{1/2}$ for solid lines have been corrected for internal conversion.
Isospin Selection Rules

Reason: n/p rearrangements → multipole charge distributions

Photon interacts only with 1 nucleon ($\tau = 1/2$) $\Rightarrow \Delta T = 0, 1$

Example $E1$ transitions:

$$\hat{O}_{E1} = \sum_p e(\vec{r}_p - \vec{R}_{cm}) = \frac{1}{2} \sum_i e\tau_{zi} (\vec{r}_i - \vec{R}_{cm}) = \text{iso-vector op}$$

Enhanced (collective) $E2, E3$ transitions $\Delta T = 0$

Collective (rot or vib) WF does not change in transition.

**Projection operator** $\hat{P}_n := \frac{1}{2} (1 - 2\tau_{zi}) = \begin{cases} 1 & p \\ 0 & n \end{cases}$

$$\hat{O}_{M1} = \frac{1}{2} \sum_i (1 - 2\tau_{zi}) \hat{l}_i + \frac{1}{2} \sum_i (1 - 2\tau_{zi}) g_p \hat{S}_i + \frac{1}{2} \sum_i (1 + 2\tau_{zi}) g_n \hat{S}_i$$

$$= \frac{1}{2} \hat{I} + \sum_i (g_n + g_p - 1) \hat{S}_i + \sum_i \tau_{zi} \left[ (g_n - g_p) \hat{S}_i - \hat{l}_i \right] \quad \text{iso–scalar}$$

$$\rightarrow 0 \quad \text{iso–vector}$$

$$g_n = -3.8 \quad g_p = +5.6$$
Gamma Decay of Isobaric Analog States

For same $T$, wfs for protons and neutrons are similar

→ “Mirror Nuclei” $^{19}\text{Ne}$ and $^{19}\text{F}$
Electrostatic Multipole (Coulomb) Interaction

\[
V(\vec{r}) = Ze \int d^3\vec{r}' \frac{e \cdot \rho(\vec{r}')}{|\vec{r}' - \vec{r}|} = \]

\[
+ \frac{Ze}{r} \int d^3\vec{r}' \rho(\vec{r}') [e \cdot 1] \]

\[
+ \frac{Ze}{r^2} \int d^3\vec{r}' \rho(\vec{r}') [e \cdot r' \cos \theta] \]

\[
+ \frac{Ze}{r^3} \int d^3\vec{r}' \rho(\vec{r}') [er'^2 \cdot (3 \cos^2 \theta - 1)] \]

\[
+ \ldots . \]

\[d^3\vec{r}' = r'^2 dr' \cdot d\phi \cdot d(\cos \theta)\]

\[\phi \text{ azimuth, } \theta \text{ polar angles} \]

\[\int d^3\vec{r}' \rho(\vec{r}') = 1\]

Quadrupole \( Q \neq 0 \), indicates deviation from spherical shape

Different multipole shapes/distributions have different spatial symmetries

Point Charges

- **Monopole** \( \ell = 0 \)
- **Dipole** \( \ell = 1 \)
- **Quadrupole** \( \ell = 2 \)

Nuclear charge distribution

\[e^2 = 1.44 \text{MeV} \cdot \text{fm}\]
**Magnetization: Magnetic Dipole Moments**

Moving charge $e \rightarrow$ current density $j \rightarrow$ vector potential $\vec{A}(\vec{r})$, influences particles at $\vec{r}$ via magnetic field $\vec{B} = \mu_0 \vec{H} = \nabla \times \vec{A}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{d^3 \vec{r}'}{|\vec{r}' - \vec{r}|} \approx \frac{\mu_0}{4\pi} \left[ \frac{1}{r} \int d^3 \vec{r}' \vec{j}(\vec{r}') + \frac{1}{r^3} \int d^3 \vec{r}' (\vec{r} \cdot \vec{r}') \vec{j}(\vec{r}') + \ldots \right]$$

$$\vec{A}(\vec{r}) \approx \frac{\mu_0}{4\pi} \frac{1}{r^3} \int d^3 \vec{r}' (\vec{r} \cdot \vec{r}') \vec{j}(\vec{r}') + \ldots = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int d^3 \vec{r}' \vec{r} \times \vec{j}(\vec{r}') \times \vec{r}' + \ldots$$

**Magnetic dipole moment**

$$\mu := \frac{1}{2} \int d^3 \vec{r}' [\vec{r}' \times \vec{j}(\vec{r}')]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{\mu} \times \vec{r}}{r^3} + \text{higher multipole moments} \rightarrow B \sim \frac{1}{r^3}$$

$$\vec{r} \times \vec{j}(\vec{r}) = \vec{r} \times \left[ \rho(\vec{r}) \frac{e}{m} \vec{p}(\vec{r}) \right]; \quad \vec{r} \times \vec{p} = \vec{L}$$

$$\mu = \frac{e}{2m} \int d^3 \vec{r}' \rho(\vec{r}') \vec{L}(\vec{r}') \bigg|_{\rho(r) = \delta(r-R)} \rightarrow \frac{e}{2m} \vec{L} \quad \text{qu.mech.} \rightarrow g_\ell \cdot \left( \frac{e\hbar}{2m} \right)_{\ell}$$

**Current loop:**

$$\mu_{\text{Loop}} = I \times A = \text{current} \times \text{Area}(\text{inside})$$
End Spectroscopy
# Lecture Plan Intro to Nuclear Structure

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<th>Day</th>
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<th>Topic</th>
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<tr>
<td>Monday 6/25</td>
<td>09:00-10:00</td>
<td>How do we know about NS? Nuclear Spectroscopy. Nucleon-Nucleon forces and 2-body Systems</td>
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<tr>
<td>Tuesday 6/26</td>
<td>09:00-10:00</td>
<td>Mean Field and its Symmetries, Spin and Isospin Fermi Gas Model</td>
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<tr>
<td>Wednesday 6/27</td>
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<td>Spherical Shell Model Simple Predictions and Comparisons</td>
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<td>Thursday 6/28</td>
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<td>Residual Interactions/ Pairing Deformed Nuclei and their Spectroscopy</td>
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<td>Friday 6/29</td>
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<td>Final Exam</td>
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