Nuclear Reactions Survey
Distinct evolution of dynamics with increasing overlap between P & T

Experimental Observables

Multiplicity of reaction products $A_i, Z_i, I_i$ of reaction products

Emission angles $\Theta_i$

Total kinetic energy $\sum_i E_{kin,i}$ vs.

Intrinsic excitation $E_i^*$

Particle correlations in any observables
Cross Section and Reaction Amplitude

Stationary approximation: Incoming plane wave = approximation to particle wave packet

\[ \psi_i \sim e^{i k_i z} = e^{i \vec{k}_i \cdot \vec{r}} ; \quad \vec{k}_i = \frac{\vec{p}_i}{\hbar} \]

\[ |\psi_i|^2 \equiv 1 \rightarrow 1 \text{ particle / volume element} \]

Outgoing wave is a spherical wave produced by the obstacle (Target)

\[ \psi_f \sim f(\theta, \varphi) \frac{1}{r} e^{i k_f r} ; \quad \vec{k}_f = \frac{\vec{p}_f}{\hbar} \]

For axial symmetry (no spins)

\[ \psi_f \sim f(\theta) \frac{1}{r} e^{i k_f r} \quad \text{quadratically integrable} \]

Detector count rate:

\[ \frac{dN_{\text{det}}}{dt} = \rho_f \cdot \nu_f \cdot dF \]

\[ = |\psi_f|^2 \nu_f \cdot dF = \nu_f \cdot \frac{dF}{r^2} \cdot |f(\theta)|^2 \]

\[ = \nu_f \cdot d\Omega \cdot |f(\theta)|^2 = \nu_f \cdot \frac{d\sigma}{d\Omega} \cdot d\Omega . \]

Scattering / Reaction Amplitude

\[ \frac{d\sigma(\theta)}{d\Omega} = |f(\theta)|^2 \]
Cross Section and Transition Probability

Small transition probabilities/time → estimate $dP_{i \rightarrow f}/dt$ by perturbation theory

Fermi's Golden Rule for transition rate (probability per time) $\dot{P}_{i \rightarrow f} : |i \rangle \rightarrow |f \rangle$

$$\dot{P}_{i \rightarrow f} \approx \frac{2\pi}{\hbar} \cdot |\langle f | \hat{H}_{\text{int}} | i \rangle|^2 \cdot \frac{dn}{dE_f}; \quad \hat{H}_{\text{int}} : \text{interaction}$$

Neglect internal structure of products: $n = n(\text{particle kinetic energies, } J_f)$

$$dn = \frac{(2J_f + 1)}{(2\pi \hbar)^3} \cdot V \cdot 4\pi p_f^2 \cdot dp_f; \quad dE_f = \frac{p_f}{m_f} \cdot dp_f; \quad J_f = \text{particle angular momentum}$$

$$\frac{dn}{dE_f} = V \cdot \frac{(2J_f + 1)4\pi}{(2\pi \hbar)^3} \cdot m_f \cdot p_f \rightarrow \text{same spatial norm. volume } V = 1 \text{ for all particle WFs}$$

$$\dot{P}_{i \rightarrow f} \approx \frac{16\pi^3}{V (2\pi \hbar)^4} \cdot |\langle f | \hat{H}_{\text{int}} | i \rangle|_{\text{norm}}^2 \cdot (2J_f + 1) \cdot m_f \cdot p_f$$

Incoming flux density: $j_i = \rho \cdot v_i \rightarrow \dot{P}_{i \rightarrow f} = j_i \cdot \sigma$

$$\sigma = \int \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{\dot{P}_{i \rightarrow f}}{j_i} \approx \frac{(2J_f + 1)}{\pi \hbar^4} \cdot |M_{if}|^2 \cdot \frac{p_f^2}{v_i \cdot v_f}$$

Matrix element $M_{if} := \langle f | \hat{H}_{\text{int}} | i \rangle_{\text{norm}}$

For better approximation use c.o.m. variables, e.g., reduced mass, relative velocities and momenta. Final state density is product of phase space density and intrinsic level densities of products
Low-Energy Cross Section Behavior

\((n, n')\) elastic scattering

\(v_i \approx v_f\) for heavy targets \(\rightarrow \sigma_{nn'}(E_n) \propto \frac{p_f^2}{v_i \cdot v_f} = \text{const.}\)

Exothermic \(n\) - induced, large \(Q\), e.g., \((n, \alpha)\)

For small \(v_i \rightarrow v_f = f(Q) \approx \text{const.} \rightarrow \sigma \sim \frac{1}{v_i}\)

Exothermic charged - part. - induced, large \(Q\), e.g., \((\alpha, p)\)

For small \(v_i \rightarrow v_f = f(Q) \approx \text{const.} \rightarrow \sigma \sim \frac{1}{v_i} e^{-\left(\frac{G_{\alpha} + G_p}{E_n}\right)} \text{(Gamov)}\)

Endothermic \(n\) - induced, \(E^* = -Q \approx 1\text{MeV}, \) e.g., \((n, \alpha)\)

\(v_f = \sqrt{\frac{(E_i - E^*)}{m_n/2}} \rightarrow \sigma \propto \frac{p_f^2}{v_i \cdot v_f} \sim v_f \propto \sqrt{E_i - E^*}\)

Threshold = \(E^*\)

Endothermic charged - Part. - induced, \(E^* = -Q \approx 1\text{MeV}, \) e.g., \((n, \alpha)\)

\(v_f = \sqrt{\frac{E_i - E^*}{m_n/2}} \rightarrow \sigma \propto \frac{p_f^2}{v_i \cdot v_f} e^{-\left(G_i + G_f\right)} \propto \sqrt{E_i - E^*} \cdot e^{-\left(G_i + G_f\right)}\)

Threshold = \(E^*\)
Thermonuclear Reactions

Environment in gaseous stars, inertial confinement fusion,... characterized by temperature $k_B T \leq 10$keV $\rightarrow$
Maxwell-Boltzmann energy distributions of reactants $\rightarrow$

Only transmission weighted high-energy tails contribute to fusion $\rightarrow$

Gamov Peak

Low – Energy Parameterization $\sigma(E) \approx \frac{1}{E} \cdot S(E) \cdot e^{-G(E)}$

Spectroscopic factor $S(E)$

When is picture of trajectories a good approximation to wave mechanics?

Short wave length limit: deBroglie wave length \( \lambda \ll \text{nucl. dimension} \)

Smooth variation of the interaction (e.g., \( V(r) \))

\[
\lambda(r) = \hbar \left( 2\mu(E - V(r)) \right)^{-1/2} \quad \mu = \text{reduced mass}
\]

\[
\lambda \cdot |\nabla V(r)| \ll V(r) \quad a \sim 1\text{fm} = \text{range of } V
\]

Large orbital angular momenta \( L \), large \( \Delta L \) sampling the force

\[
\Delta L = \hbar k \cdot a \gg \hbar |\nabla V(r)|/V
\]

Heavy – ion reactions, e.g., \(^{40}\text{Ca} + ^{124}\text{Sn} @ E \sim \text{GeV} \)

\( \lambda \sim 10^{-2}\text{fm}, \quad \Delta L \sim 1k\hbar \)

→ Provide intuitive understanding of observable reaction phenomena