Experimental Chart of Nuclides 2000
2975 isotopes

Gross Properties of Nuclei
Nuclear Masses and Binding Energies
Measuring Nuclear Masses

Used with ISOL target to measure exotic reaction product nuclei

New nuclides are produced in high charge ($q$) states $\rightarrow$ good mass resol.

**Lorentz force**

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\rightarrow$ **Wien velocity Filter**

\[
B \rho_B \propto \frac{mv}{q} \\
E \rho_E \propto \frac{K}{q} \\
\rightarrow \nu, m, K
\]

Abs: $\frac{\Delta m}{m} \approx 10^{-3}$

Future: $\frac{\Delta m}{m} \approx 10^{-5}$ | Rel: $\approx 10^{-8}$
TRIUMF-Dragon Recoil Mass Spectrometer
Basic Constituents of Atomic Nuclei

Helium

\[ \frac{A}{Z} X_N = \frac{4}{2} He_2 \]

- \( A \) nucleons: protons (H\(^+\)) plus neutrons
- \( Z \) protons (equals the number of electrons in a neutral atom)
- \( N = A - Z \) neutrons

\[ \frac{A}{Z} X_N \rightarrow= \frac{A}{X} X \] (ex: \( ^4 He \))

{Briefly at \( E^* \gg 0 \) also (virtual) \( \pi, \Delta, \ldots \)}

Charges

- \( e_p = +e = 1.602 \cdot 10^{-19} C \) (Coulomb)
- \( e_n = 0 \) \((e^2=1.440\text{MeVfm})\)

Masses

- \( m_p = 1.673 \cdot 10^{-27} \text{kg} = 938.279 \text{MeV/c}^2 = 1.00728 \text{u} \quad \Delta m_p = 7.289 \text{MeV rel}^{12}C \)
- \( m_n = 1.675 \cdot 10^{-27} \text{kg} = 939.573 \text{MeV/c}^2 = 1.00867 \text{u} \quad \Delta m_n = 8.071 \text{MeV rel}^{12}C \)

\( 1\text{u} = m^{(12}C)/12 = 1.6606 \cdot 10^{-27} \text{kg} = 931.502 \text{ MeV/c}^2 = \text{standard nucleonic mass} \)

Expect approximately:

\[ m\left(\frac{A}{Z} X_N\right) = Z \cdot m_p + N \cdot m_n + Z \cdot m_e \]

Find:

\[ m\left(\frac{A}{Z} X_N\right) < Z \cdot m_p + N \cdot m_n + Z \cdot m_e \]

\[ \Delta m \cdot c^2 = B \]

Mass Defect
Radiative Capture of Nucleons

Elastic NN scattering ($E \geq 0$) does not lead to a bound ($E < 0$) NN system (nucleus).

Need to “dissipate” extra energy \(\rightarrow\) radiate out some of the mass as \(\gamma\)-ray.

Hypothetical nuclear “condensation”: \(E_\gamma\), radiated as \(\gamma\)-rays lost from nucleus \(\rightarrow\) less energy than original free nucleons

\[
\Delta E < 0,\quad \Delta E + E_\gamma = 0
\]

Observe radiation emitted in capture of \(n, p, \ldots, e^-\)
### Nuclear Masses

\[ mc^2 = \sum_i m_i c^2 \]  

- **Binding energy** \( B \)

\[
B = \left[ Z \cdot m_H + N \cdot m_n \right] - m \left( \frac{A}{Z} \right) X_N > 0 \]

<table>
<thead>
<tr>
<th>Element</th>
<th>Mass of Nucleons (u)</th>
<th>Nuclear Mass (u)</th>
<th>Binding Energy (MeV)</th>
<th>Binding Energy MeV/Nucleon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuterium</td>
<td>2.01594</td>
<td>2.01355</td>
<td>2.23</td>
<td>1.12</td>
</tr>
<tr>
<td>Helium 4</td>
<td>4.03188</td>
<td>4.00151</td>
<td>28.29</td>
<td>7.07</td>
</tr>
<tr>
<td>Lithium 7</td>
<td>7.05649</td>
<td>7.01336</td>
<td>40.15</td>
<td>5.74</td>
</tr>
<tr>
<td>Beryllium 9</td>
<td>9.07243</td>
<td>9.00999</td>
<td>58.13</td>
<td>6.46</td>
</tr>
<tr>
<td>Iron 56</td>
<td>56.44913</td>
<td>55.92069</td>
<td>492.24</td>
<td>8.79</td>
</tr>
<tr>
<td>Silver 107</td>
<td>107.86187</td>
<td>106.87934</td>
<td>915.23</td>
<td>8.55</td>
</tr>
<tr>
<td>Iodine 127</td>
<td>128.02684</td>
<td>126.87544</td>
<td>1072.53</td>
<td>8.45</td>
</tr>
<tr>
<td>Lead 206</td>
<td>207.67109</td>
<td>205.92952</td>
<td>1622.27</td>
<td>7.88</td>
</tr>
<tr>
<td>Polonium 210</td>
<td>211.70297</td>
<td>209.93683</td>
<td>1645.16</td>
<td>7.83</td>
</tr>
<tr>
<td>Uranium 235</td>
<td>236.90849</td>
<td>234.99351</td>
<td>1783.8</td>
<td>7.59</td>
</tr>
<tr>
<td>Uranium 238</td>
<td>239.93448</td>
<td>238.00037</td>
<td>1801.63</td>
<td>7.57</td>
</tr>
</tbody>
</table>

**Note:** hydrogen atomic mass \( m_H \) is used in tables, rather than \( m_p \).
Systematics of Experimental Binding Energies

For heavy nuclei, $B/A \approx 8\text{MeV}$

**Light nuclei:**

- odd-even staggering

- $gg$ nuclei are most tightly bound
- $uu$ nuclei least bound

**Exceptions:**

- $^8\text{Be}$ unstable, $^8\text{Be} \rightarrow 2\, ^4\text{He}$

- There is no stable $A=5$ nucleus.

$^4\text{He}, (\ ^8\text{Be}), ^{16}\text{O}, ^{20}\text{Ne}, \ldots$
Energetics of the A=8 System

Systems with less BE energetically allowed to transform (decay) into more strongly bound systems

\[ \Delta m^2_{12C} (B) \]

\[ 4n + 4p \quad \Delta m = 4 \cdot (8.071 + 7.289) \text{MeV} \quad \rightarrow 61.44 \text{MeV} \]

per def. \( B = (61.44 - 61.44) \text{MeV} = 0 \text{MeV} \)

\[ ^8\text{Be} \quad \Delta m = 4.942 \text{MeV} \quad \rightarrow B = (61.44 - 4.942) \text{MeV} = 56.49 \text{MeV} \]

\[ -7.06 A \text{MeV} \]

\[ 2\alpha \quad \Delta m = 2 \cdot 2.425 \text{MeV} \quad \rightarrow B = (61.44 - 4.85) \text{MeV} = 56.59 \text{MeV} \]

\[ -7.07 A \text{MeV} \]

\[ ^8\text{Be} \text{ bound but unstable (} \Delta E \approx 100 \text{keV} \text{) against decay } \rightarrow 2 \alpha \]

\[ t_{1/2} = 7 \cdot 10^{-17} \text{s} \]
The Semi-Empirical Mass Formula

\[ m(Z,N) = [Z \cdot m_H + N \cdot m_n] - B(Z,N)/c^2; \quad B > 0 \]

\[ B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} - \delta A^{-1/2} \]

\( a_v = 15.835 \text{ MeV} \)
\( a_s = 18.33 \text{ MeV} \)
\( a_c = 0.714 \text{ MeV} \)
\( a_a = 23.20 \text{ MeV} \)

\( a_v \) = volume, “condensation”
\( a_s \) = surface, less binding
\( a_c \) = Coulomb repulsion of protons
\( a_a \) = asymmetry lessens binding for like particles, compared to unlike
\( \delta \) = “pairing,”
unpaired particles less tightly bound

Implicit assumption:
“Leptodermous” structure-less nucleus

\[ \delta = \begin{cases} +11.2 \text{ MeV for } o-o \text{ nuclei} \\ 0 \text{ MeV for odd- } A \text{ nuclei} \\ -11.2 \text{ MeV for } e-e \text{ nuclei} \end{cases} \]
Relative Contributions to Nuclear Mass

\[ R \propto A^{1/3} \rightarrow V_{\text{nucleus}} \approx A \]

const. contribution from each nucleon \( \rightarrow \) “saturated” force

fewer interactions on surface \( \rightarrow \)
reduce contribution from each surface nucleon \( S \propto A^{2/3} \)

different interactions between like and unlike nucleons (Fermion statistics, isospin) \( \rightarrow \)
depends on \( |N-Z| \), reduces B

Coulomb self energy becomes large for large \( Z \), heavy nuclei, makes nucleus unstable
reduces B

\[ E_{\text{Coul}} = \frac{e^2 Z^2}{2} \int d^3\vec{r} d^3\vec{r}' \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} = \frac{3}{5} \frac{e^2 Z^2}{R_C} \]

\[ R_C = 1.24 A^{1/3} \text{fm} \]

hom. sharp sphere
### Energetics of Transmutation

Non-monotonic behavior,\

\[
BE \propto A
\]

Fe/Ni most strongly bound

2 different regions in \( A \), different energetically preferred transmutations:

#### Heavy nuclei are unstable, exothermic spontaneous emission of \( \alpha \) particles (\( B_\alpha = 28 \text{ MeV!} \))

#### Heavy nuclei can split ("fission") \( \rightarrow \) nuclear power.

2 light nuclei can fuse and produce energy (stars, nuclear power)

**Mass defect**

\[
m_{Ra} - m_{RnHe} = 0.0052 \text{ u}
\]

\( \rightarrow \) exothermic,

energy released \( Q = +4.84 \text{ MeV} \)

Transmutation energetically possible

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**Example Transmutation**

\[
^{226}\text{Ra} \rightarrow (^{222}\text{Rn} + ^{4}\text{He})
\]

(226.0254 MeV) \( \rightarrow \) (222.0176 + 4.00260) MeV

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**Graph**

- Binding energy per nucleon (MeV) vs. Mass number (A)
- Energy released by fusion
- Energy released by fission

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**Equation**

\[
^{226}\text{Ra} \rightarrow (^{222}\text{Rn} + ^{4}\text{He})
\]

(226.0254 MeV) \( \rightarrow \) (222.0176 + 4.00260) MeV

\[
m_{Ra} - m_{RnHe} = 0.0052 \text{ u}
\]

\( \rightarrow \) exothermic,

energy released \( Q = +4.84 \text{ MeV} \)

Transmutation energetically possible
The Surface Symmetry Energy Component

\[ m(Z, N) = [Z \cdot m_H + N \cdot m_n] - B(Z, N)/c^2; \quad B > 0 \]

neglect e⁻ binding


\[ B(N, Z) = a_V A - a_s A^{2/3} - a C \frac{Z^2}{A^{1/3}} - a_{Vs} \frac{(N - Z)^2}{A} + a_{Ss} \frac{(N - Z)^2}{A^{4/3}} - \delta A^{-1/2} \]

Refit data with 2 symmetry energy terms:
- \( a_V = 15.7 \text{ MeV} \)
- \( a_S = 18.1 \text{ MeV} \)
- \( a_{Vs} = 26.5 \text{ MeV} \)
- \( a_{Ss} = 19.0 \text{ MeV} \)
- \( \delta = \begin{cases} +12.8 \text{ MeV for o–o nuclei} \\ 0 \text{ MeV for odd–Anuclei} \\ -12.8 \text{ MeV for e–e nuclei} \end{cases} \)

“Symmetry energy” results from quantal Pauli Exclusion Principle \( \rightarrow \) density \( (\rho) \) dependent

Surface has lower average density than volume \( \rightarrow \) symmetry energy lower in surface (note the different signs !)

Fits: \( a_{Vs} \) and \( a_{Ss} \) are strongly correlated
\( \rightarrow \) difficult to determine them independently

**Surface symmetry energy has x2 uncertainty!**

\((N-Z)^2\) dependence of symmetry terms comes from semi-classical expansion for large \( A \).

Stability: Coulomb vs. Symmetry Energy

Light nuclei with few nucleons:
Few isotopes, \( N \approx Z \approx A/2 \)

Medium-weight to heavy nuclei:
Binding energy \( BE(Z) \) is smooth function of \( Z \rightarrow \) average ("gross") behavior of nuclei most important

\( BE(Z)_{A=\text{const}} \) is non-monotonic:
For any \( A \), there is a maximum \( BE \) for a characteristic \( Z(A) < A/2 \)

NOT: smallest charge densities, Nuclei with large \( N \) are also unstable

→correlated values of \( Z \) and \( N \) (neutrons and protons) Quantum Pauli correlations

→But high Coulomb self-energy shifts max binding energy/\( A \) to higher \( N \).

The Odd-Even (Pairing) Effect of Binding Energies

Even-$A \rightarrow$ even-$N +$ even-$Z$ (e e) or odd-$N +$ odd-$Z$ (o o)  
Effect not visible for odd-$A$ nuclei

e-e nuclei have slightly, but systematically, stronger binding  
than o-o neighbors
Structure Effects in the Pairing Energies

odd-even mass differences
B largest for “paired” nucleons

Average trend:

$$B_{LD} - B_{expt} = \Delta(N, Z)$$

$$\Delta_n \approx \Delta_p \approx \delta A^{-1/2}$$

Remaining structure and fluctuations: Effect due to intrinsic nuclear structure (“shell model”) and collective deformation.

Weak indications of special structure at “magic” neutron and/or proton numbers ($N, Z = 8, 20, 28, 50, 82, 126,...$): several isotopes/isotones
Experimental Chart of Nuclides 2000
2975 isotopes

Nature:
170 even-Z, even-N
60 odd-A
4 odd z, odd N

\[ Z_A \approx \frac{A}{2} \]

The β-Stable Valley

\[ Z_A \approx \frac{A}{2 + A^{2/3} a_c/(2a_{sym})} \]
Droplet Model


Extension of LDM → 1 higher order in $A^{-1/3}$, $I=(N-Z)/A$

finite compressibility, deformed shapes, $\rho_n$ and $\rho_p$ different surfaces.

Most accurate: **Finite-Range Droplet Model** (also: most parameters)

$$B(N, Z; \text{shape}) = \left[ -a_1 + J\bar{\delta}^2 - \frac{1}{2} K\bar{\varepsilon}^2 + \frac{1}{2} M\bar{\delta}^4 \right] A + \left[ a_2 + \frac{9}{4} \frac{J^2}{Q} \bar{\delta}^2 \right] A^{2/3} B_{\text{surf}}$$

$$+ a_3 A^{1/3} B_{\text{curv}} + c_1 Z^2 A^{-1/3} B_C - c_2 Z^2 A^{1/3} B_{\text{red}} - c_5 Z^2 B_w - c_3 Z^2 A^{-1} - c_4 Z^{4/3} A^{-1/3}$$

$$\bar{\delta} = \frac{I + (3c_1/16Q)ZA^{-2/3}B_v}{1 + (9J/4Q)A^{-1/3}B_{\text{surf}}}; \quad \bar{\varepsilon} = \left[ -2a_2 A^{-1/3} B_{\text{surf}} + L\bar{\delta}^2 + c_1 Z^2 A^{-4/3} B_C \right] / K$$

bulk symmetry \hspace{1cm} deviation from average density

$$c_1 = \frac{3e^2}{(5r_0)}; \quad c_2 = \frac{c_1^2}{336} \left( \frac{1}{J} + \frac{18}{K} \right); \quad c_3 = \frac{5c_1}{2} \left( \frac{b}{r_0} \right)^2;$$

$$c_4 = \frac{5c_1}{4} \left( \frac{3}{2\pi} \right)^{2/3}; \quad c_5 = \frac{1}{64} \frac{c_1^2}{Q} \quad \text{Functions B describe contributions of shape/spherical}$$
Quality of Droplet-Model Mass Fit


Total binding energy of heavy nuclei ~1600 MeV, accuracy of LDM fit: ±0.5 MeV