Nuclear s.p. Models
The Deformed Shell Model
Nuclear quadrupole moments \( (Q_0) \) are large for \( N, Z \) between “magic” numbers. Close to magic numbers nuclei are spherical \( (Q_0 \approx 0) \), small and tightly bound.
Rotational symmetry broken \(\rightarrow\)  
Conserved quantities: nucleonic angular momentum projections on symmetry axis:

\[
\Lambda = \text{projection of orbital } \vec{\ell} \\
\Sigma = \pm 1/2 = \text{projection of spin } \vec{S} \\
\Omega = \Lambda + \Sigma = \text{projection of total angular angular momentum } \vec{\ell} + \vec{S}
\]

Energy level degeneracy:  
\(D_\Omega = 2 \ (\pm \Omega) \text{ axial symmetry.}\)

Anisotropic harmonic oscillator model \(\rightarrow\) Principal quantum number  
\(N = n_x + n_y + n_z = \) number of oscillator quanta.  
\((\omega_x, \omega_y, \omega_z) \text{ are different fundamental frequencies}\)
Anisotropic Harmonic Oscillator Model

Quantum Numbers \([N, n_z, \Lambda, |\Omega|]\)

\[
V(\bar{r}) = \frac{m}{2} \left( \omega_z^2 \cdot z^2 + \omega_{x,y}^2 \left( x^2 + y^2 \right) \right) 
\]

\[
V(r, \theta) \approx \frac{m}{2} \omega_0^2 \cdot r^2 \left[ 1 - \frac{4}{3} \delta \cdot P_2 (\cos \theta) \right]
\]

\(\theta : \angle \bar{r}, \hat{z}\) axis

Deformation parameter \(\delta\)

\[
\omega_z \approx \omega_0 \left( 1 - \frac{2}{3} \delta \right) \quad \omega_{x,y} \approx \omega_0 \left( 1 + \frac{1}{3} \delta \right)
\]

Volume conservation: \(\omega_0^3 = \omega_z \cdot \omega_{x,y}^2\)

\(N = n_x + n_y + n_z \rightarrow\)

\[
E_N = (n_z + 1/2) \hbar \omega_z + (n_x + n_y + 1) \hbar \omega_{x,y}
\]

Each level carries up to \(D_\Omega = 2\) nucleons

\[
\Lambda = |n_{x,y}|, |n_{x,y}| - 2, |n_{x,y}| - 4, \ldots \]

Modified Nilsson Potential (flatter potential)

\[
V(r, \theta) \approx \frac{m}{2} \omega_0^2 \cdot r^2 \left[ 1 - \frac{4}{3} \delta \cdot P_2 (\cos \theta) \right] + C \vec{\ell} \cdot \vec{s} + D \cdot \vec{\ell}^2
\]

Ground state spin determined by last (unpaired) particle.
Shell Stabilized Deformation

Why are not all nuclei spherical?
Observation: As nucleons are added to spherical (magic, closed-shell) nuclei, deformation increases → microscopic origin in nuclear structure & stability.
Approximate trends:
\[ \bar{\sigma} := \frac{1}{3} (\omega_x + \omega_y + \omega_z) = \frac{1}{3} (2\omega_{x,y} + \omega_z) \]
\[ \delta := \frac{\omega_{x,y} - \omega_z}{\bar{\sigma}} = 3 \frac{\omega_{x,y} - \omega_z}{2\omega_{x,y} + \omega_z} \rightarrow \]
\[ E_N = \hbar \omega_z (n_z + 1/2) + \hbar \omega_{x,y} (n_{x,y} + 1) \]
\[ E_N(\delta) = \hbar \bar{\sigma} \left[ \left( N + \frac{3}{2} \right) - (n_z - n_{x,y}/2) \cdot \delta \right] \]

Increasing \( \delta \rightarrow \) larger \( n_z \)
/smaller \( n_{x,y} \) levels bind more strongly. \( \rightarrow \) Shell mixing.
New shells at integer axis ratio. \( \rightarrow \) Disappearance and reappearance of shells.

After G. Musiol et al., Kern-u. Elementarteilchenphysik, VCH 1988
W. Udo Schröder, 2012
End Def. Shell Model