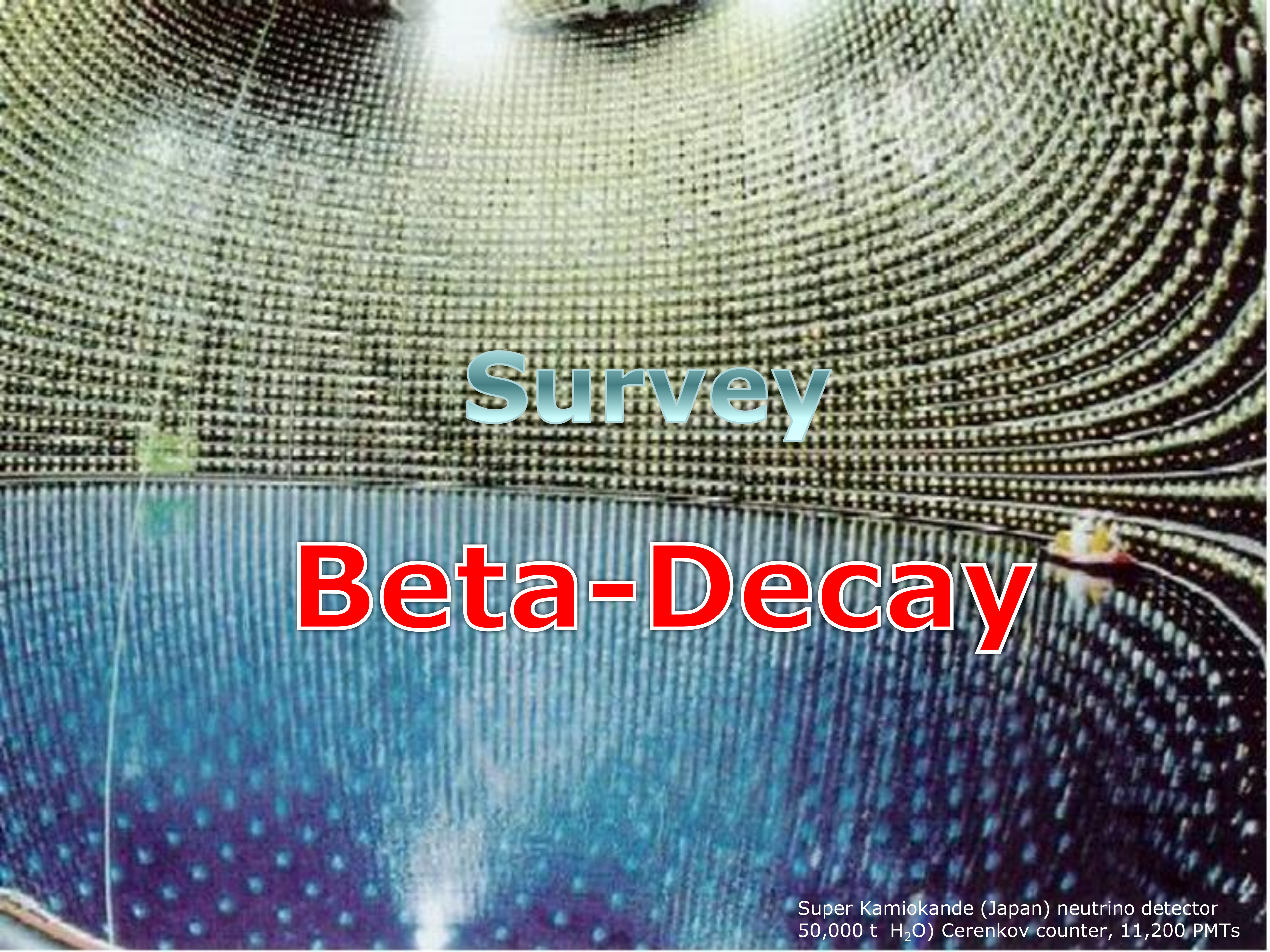
Survey

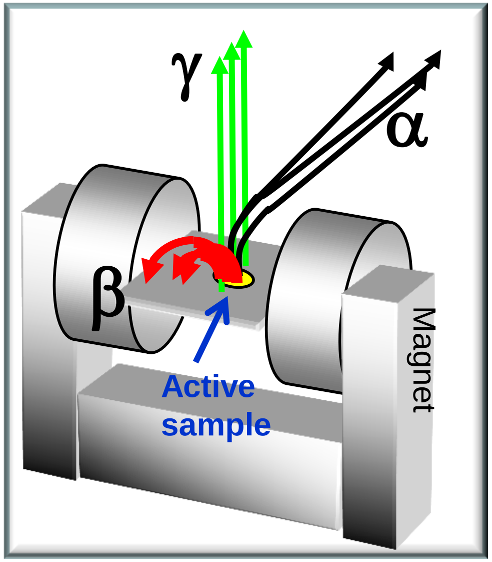
Beta-Decay



Super Kamiokande (Japan) neutrino detector
50,000 t H_2O) Cerenkov counter, 11,200 PMTs

Electron/Beta Spectrometry

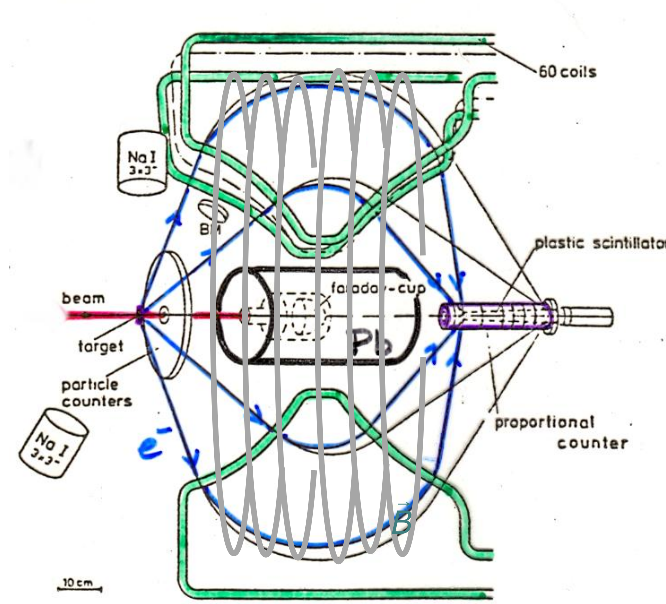
Chadwick (1914): Some nuclides emit e- with continuous energy spectra "β rays"



Radioactive Ra sample in a magnetic field β = **e⁻**.

Observed later in decay of neutrons and excited nuclei (internal conversion) or nuclear transmutation (β decay).

Iron-free "Orange" spectrometer with axially symmetric toroidal magnetic field inside current loops



60 Helmholtz coils every 6^0 arranged in a circle.
Current: ~1000 A

Setup used in nuclear reaction studies (counters for coincident particles & γ-rays)

Different energies correspond to different locations on focal detector

Circular $e^{\pm}$ orbit radius ρ in $\vec{B}$ field

$$p_e = e \cdot B \cdot \rho$$

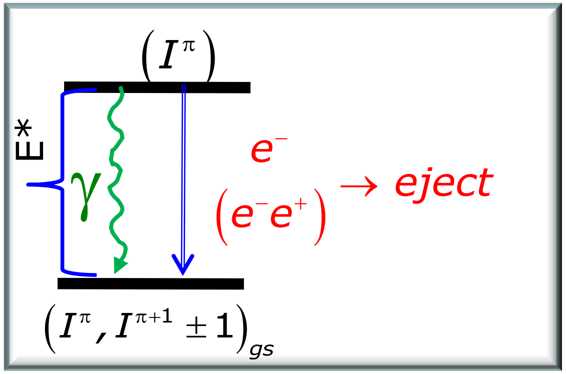
$$E_e = p_e^2 / 2m_e \rightarrow dE_e = \left(p_e / m_e\right) dp_e$$

$$\frac{dN}{dE_e} = \left(\frac{m_e}{p_e}\right) \cdot \frac{dN}{dp_e}$$

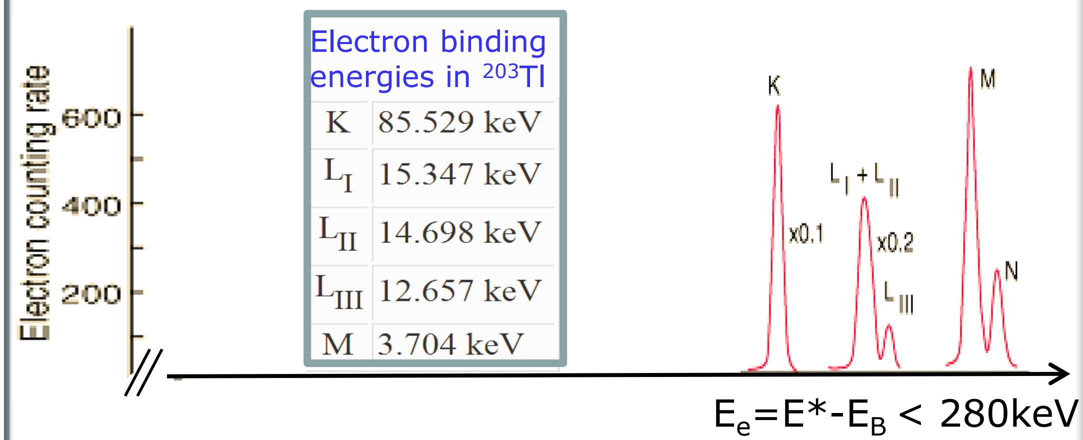
Energy spectrum constructed from momentum spectrum

Electron and Beta Spectroscopy

Nuclei can deexcite via photon, (e^+, e^-) , or atomic-electron emission (internal conversion)



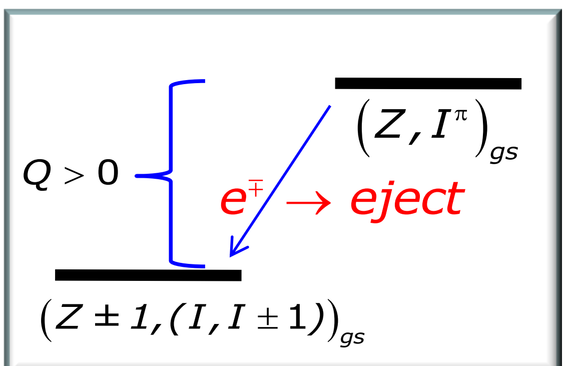
Conversion electron line spectrum for decay of ^{203}Tl state E*=280-keV



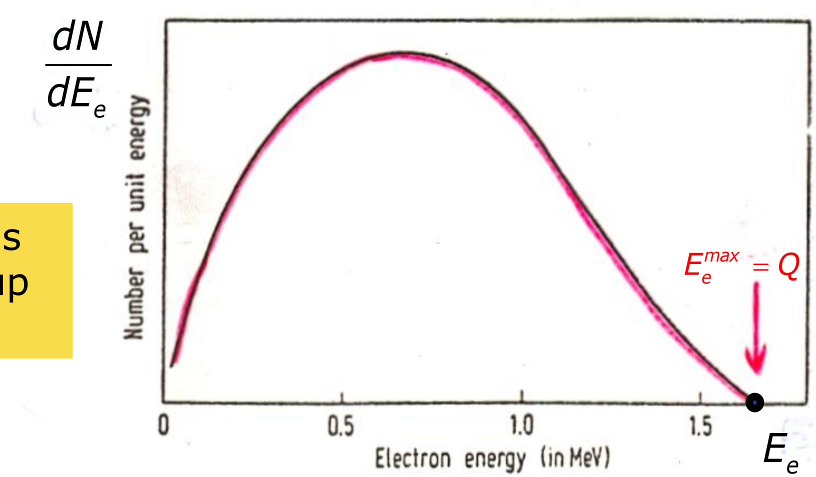
Electron binding energies in ^{203}Tl

K	85.529 keV
L_I	15.347 keV
L_{II}	14.698 keV
L_{III}	12.657 keV
M	3.704 keV

Nuclei transmute in β decay

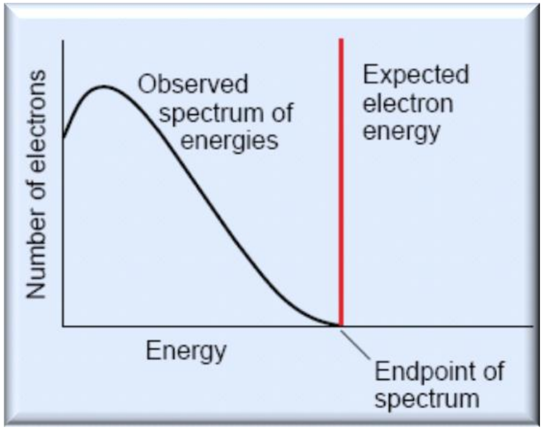


β spectrum is continuous up to $E_e \approx Q$



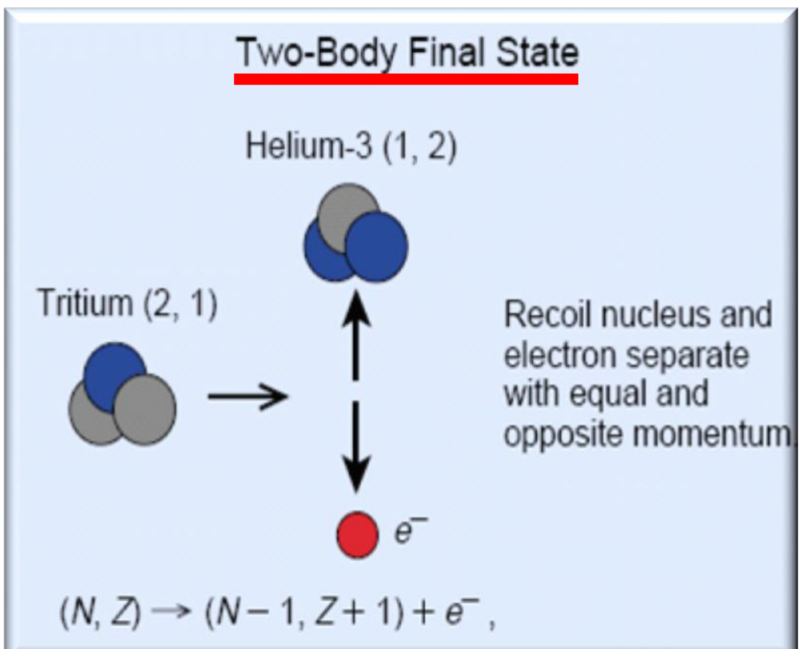
Fixed differences Q and |ΔI| carried by more than one decay product → additional "neutrinos" $\nu, \bar{\nu}$

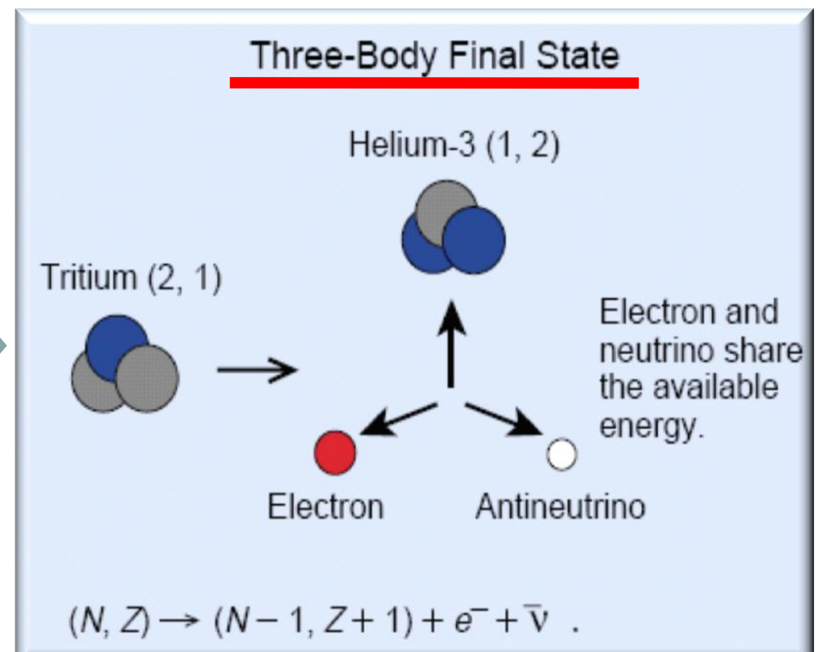
The Neutrino Hypothesis



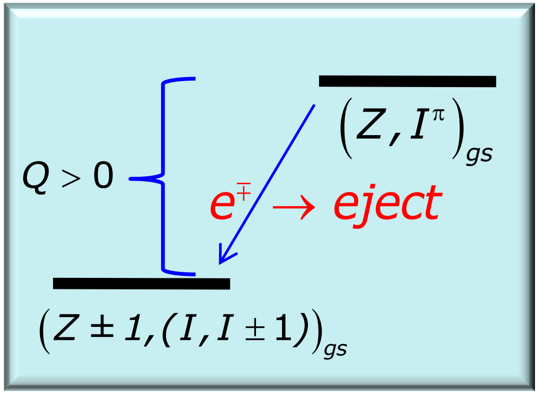
Dilemma: continuous e- spectrum would violate energy/momentum balance in 2-body process.

Wolfgang Pauli (1930) postulates unobserved, neutral particle ("neutron" later ="neutrino" (Fermi))

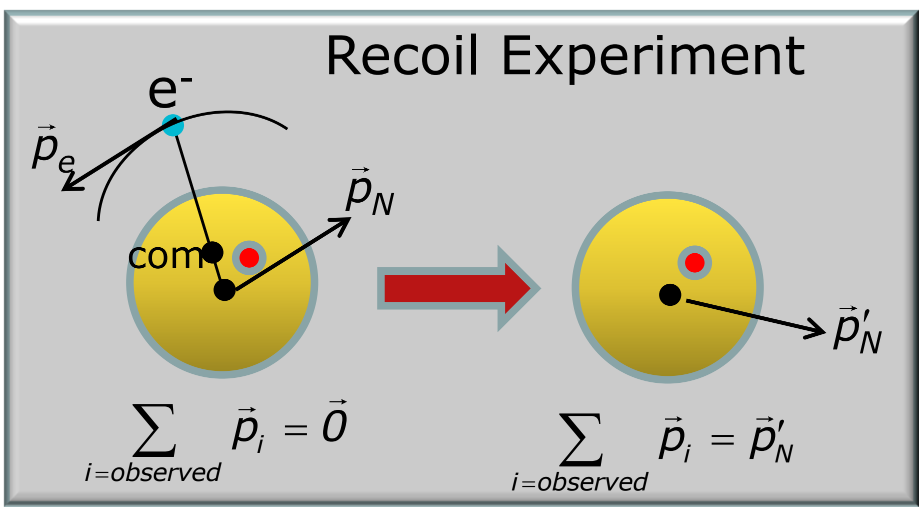




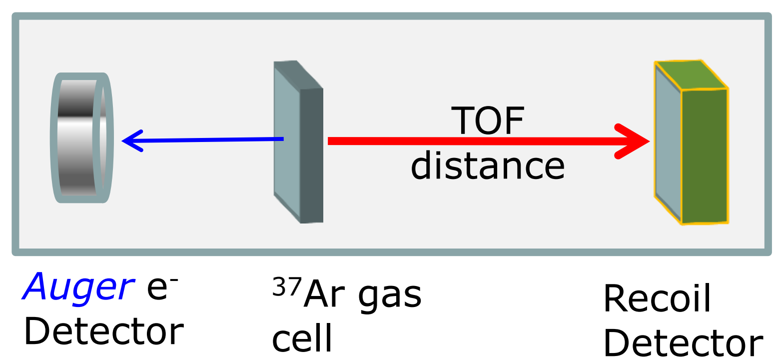
Evidence for Neutrino



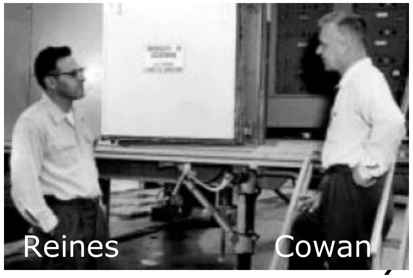
- Fixed decay energy (Q value ←→ Δmc^2) but continuous e^- spectrum
- e- has spin $I_e = 1/2$ but $|I_{final} - I_{in}| = 0, 1$ typically
- Electron capture produces recoil momentum
- Direct evidence by neutrino-induced reaction



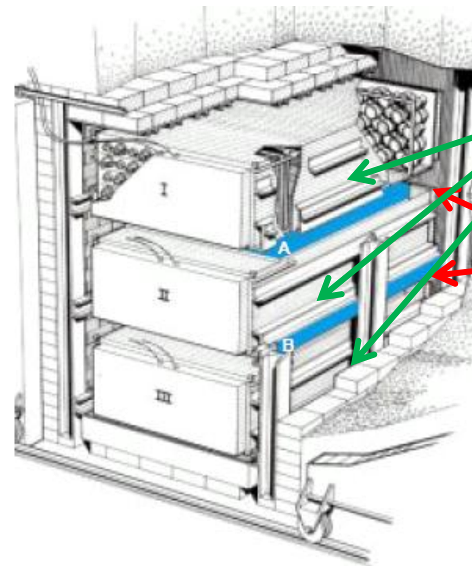
$$^{37}Ar + e^- \rightarrow {}^{37}Cl + \nu$$



Direct Evidence for Neutrino



Savannah River
reactor experiment
(fission fragments decay→$\bar{\nu}$
900 hrs with reactor on
250 hrs reactor off



$\bar{\nu}_e + p \rightarrow e^+ + n$

Experiment: $\sigma = 7 \cdot 10^{-19} b$

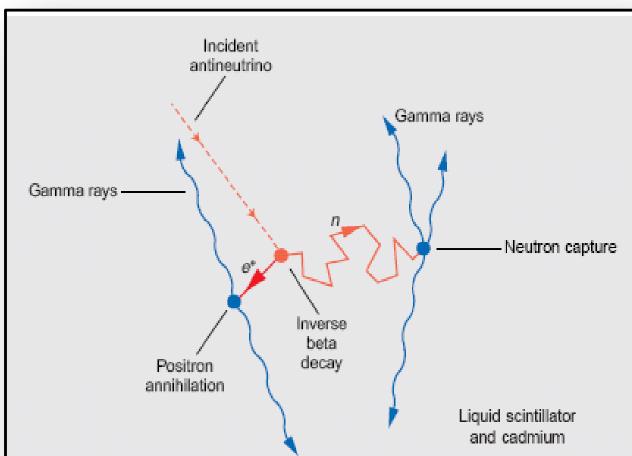
e^+/e^- annihilation

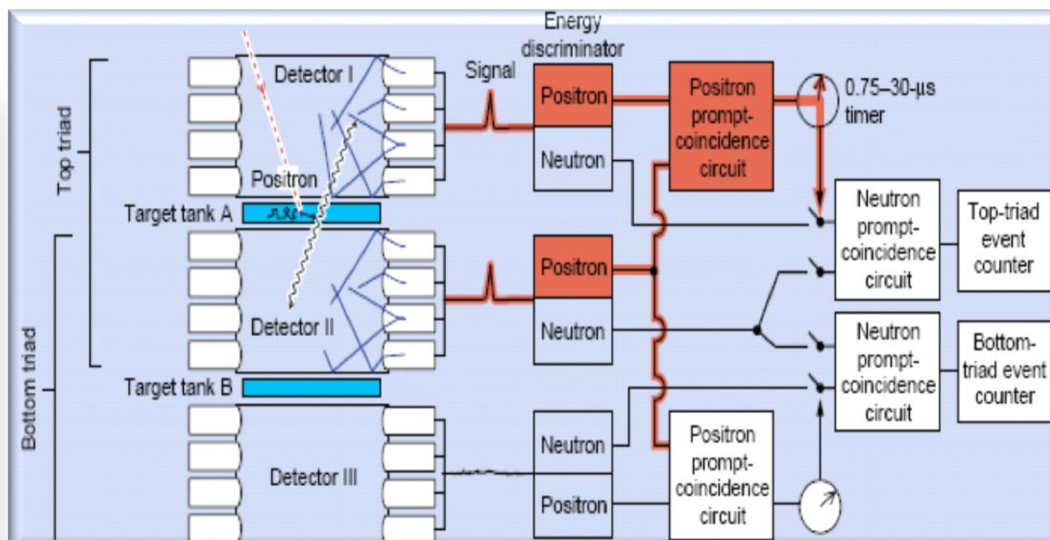
$e^+ + e^- \rightarrow 2\gamma(511 keV)$

Delayed $n-capture\ \gamma-rays$

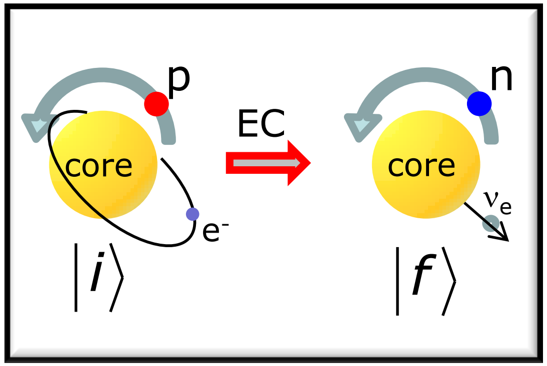
$n_{th} + {}^{109}Cd \rightarrow {}^{110}Cd^* \rightarrow {}^{110}Cd + x\gamma$

prompt e⁺-delayed capture γ coincidences





Fermi Theory of β Decay



Simple example: single nucleon orbiting core of paired nucleons captures atomic 1s electron.

Isospin wave functions χ_p , χ_n

Isospin operators $\hat{\tau}^2$, $\hat{\tau}_3$, $\hat{\tau}_\pm$ *analog to spin operators*

$$\hat{\tau}_3\chi_n = +(1/2)\,\chi_n \qquad \hat{\tau}_3\chi_p = -(1/2)\,\chi_p$$

$$\hat{\tau}_+\chi_p = \chi_n \qquad \hat{\tau}_-\chi_n = \chi_p$$

$|i\rangle \rightarrow \psi_p = \psi(\vec{r})\,\chi_p \qquad |f\rangle \rightarrow \psi_n = \psi(\vec{r})\,\chi_n$ initial, final s.p. nuclear states

$$P_{if} = \frac{2\pi}{\hbar}\left|\langle f|\hat{H}_{WI}|i\rangle\right|^2 \cdot \rho(E_f)$$

Fermi's Golden Rule (Pauli)

1st order "Perturbation Theory" for $i \rightarrow f$

ME of weak interaction *H*

Density of final states per unit energy

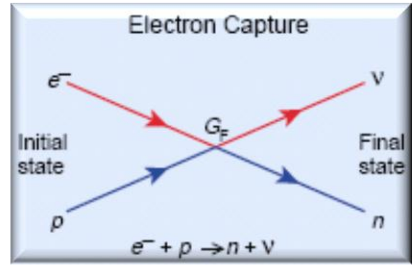
Weak Interaction Hamiltonian (point-like)

$$\hat{H}_{WI} = G_F \cdot \hat{\tau}_+ \cdot \delta(\vec{r}_p - \vec{r}_e) \cdot \delta(\vec{r}_n - \vec{r}_\nu) \cdot \delta(\vec{r}_p - \vec{r}_n) \sim G_F$$

G_F: coupling constant,
$\hat{\tau}_+$: Isospin raising operator
δ : delta distribution

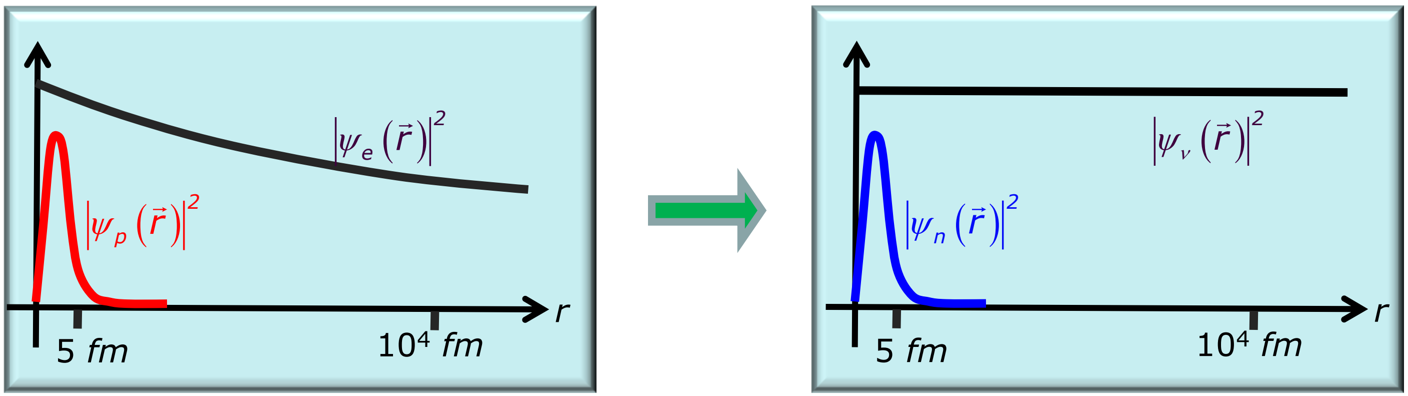
Weak Transition Matrix Elements

$$H_{fi} := \langle f | \hat{H}_{WI} | i \rangle = \int d^3\vec{r}\, \psi_f^*(\vec{r}) \cdot G_F \hat{\tau}_+ \cdot \psi_i(\vec{r})$$



$$\psi_i(\vec{r}) = \psi_e(\vec{r})\,\psi_p(\vec{r})\,\psi_{Nucl\,core}(\vec{r})$$

$$\psi_f(\vec{r}) = \psi_\nu(\vec{r})\,\psi_n(\vec{r})\,\psi_{Nucl\,core}(\vec{r})$$



Lepton wave functions vary weakly over nuclear volume →

$$|\langle \psi_{e,\nu}(\vec{r}) \rangle|^2 \approx |\psi_{e,\nu}(0)|^2$$

$$|H_{fi}|^2 = G_F^2 \left| \int_{Nucl} d^3\vec{r}\, \psi_f^*(\vec{r})\, \hat{\tau}_+ \psi_i(\vec{r}) \right|^2 \approx$$

$$\approx G_F^2\, |\psi_e(0)|^2\, |\psi_\nu(0)|^2\, \underbrace{|\langle \psi_{Nucl\,core} | \psi_{Nucl\,core} \rangle|^2}_{=1,\ \text{per def}}\, \underbrace{\left| \int_{Nucl} d^3\vec{r}\, \psi_n^*(\vec{r})\, \hat{\tau}_+ \psi_p(\vec{r}) \right|^2}_{=1 (\text{allowed tr})}$$

Fermi Transition ME

$$\left|H_{fi}\right|^2 \approx G_F^2 \left|\psi_e(0)\right|^2 \left|\psi_\nu(0)\right|^2$$

Hydrogen-like e^- wave function

$$\left|\psi_e(0)\right|^2_{1s} = 2 \cdot \frac{Z^3}{\pi a_B^3} \cdot e^{-\frac{2Zr}{a_B}}$$

$$Bohr\ Radius\ a_B = 5 \cdot 10^4 fm$$

Plane-wave ν_e wave function

$$\psi_\nu(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$$

Normalization volume, drops out in final calculations

$$\left|\psi_\nu(0)\right|^2 = \frac{1}{V}\left|e^{i\vec{k}\cdot\vec{r}}\right|^2 = \frac{1}{V}$$

$$\left|H_{fi}\right|^2 \approx G_F^2 \frac{2 \cdot Z^3}{\pi a_B^3} \cdot \frac{1}{V}$$

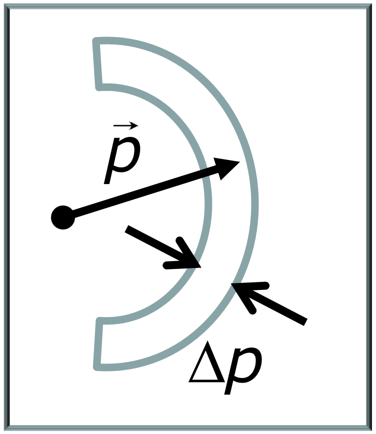
Fermi transitions ("super-allowed"): No change in I, π

For P_{if} need to evaluate density $\rho(E_f)$ of final states: neutron-neutrino relative phase space

Neutrino Phase Space

$$P_{if} = \frac{2\pi}{\hbar} G_F^2 \frac{2 \cdot Z^3}{\pi a_B^3} \cdot \frac{1}{V} \cdot \rho(E_f)$$

ρ=# final (n, ν) states at energy E_f EC:
$E_f \approx E_\nu$ neglect nuclear recoil energy



$$\underbrace{\left(\Delta p_x \cdot \Delta p_y \cdot \Delta p_z\right)}_{4\pi p^2 dp} \cdot \underbrace{\left(\Delta x \cdot \Delta y \cdot \Delta z\right)}_{dV} \geq h^3$$

Uncertainty Relation

$$d^2 n_\nu = 4\pi p_\nu^2 dp_\nu \, dV / h^3 \quad ; \quad p_\nu \approx E_\nu / c$$

$$\rho(E_f) = \frac{dn_\nu}{dE_\nu} = \frac{E_\nu^2}{2\pi^2 \hbar^3 c^3} V$$

$$P_{if} = \frac{2\pi}{\hbar}\left[G_F^2 \frac{2 \cdot Z^3}{\pi a_B^3} \cdot \frac{1}{V}\right] \cdot \left[\frac{E_\nu^2}{2\pi^2 \hbar^3 c^3} V\right] = G_F^2 \frac{2 \cdot Z^3}{\pi^2 \hbar^4 c^3 a_B^3} E_\nu^2 =: \lambda_{gs}$$

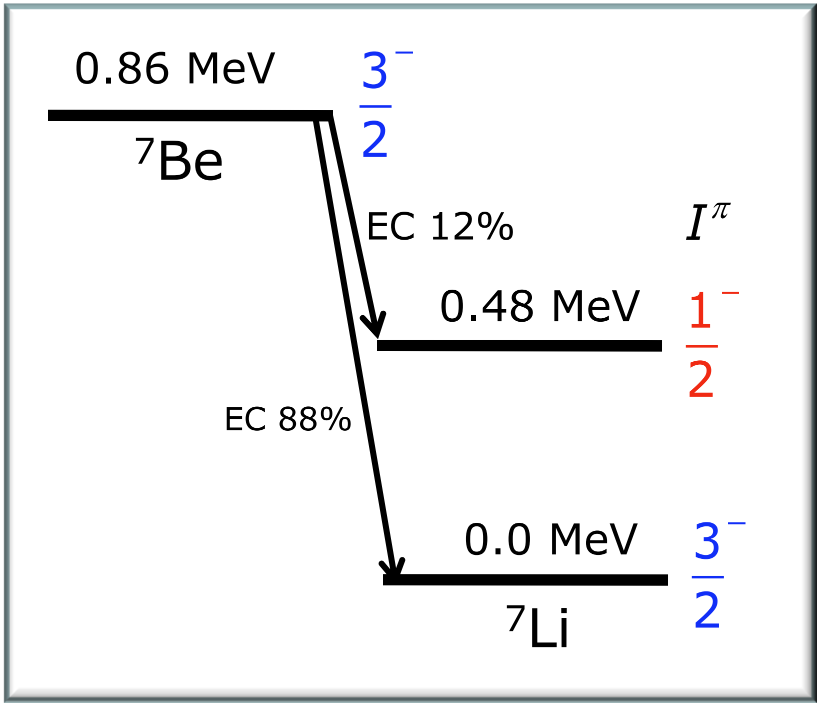
Use experimental data for ^{7}Be EC decay to determine G_F →
$G_F \approx 100$ eV fm^3. More exact average over many data sets:
$G_F \approx$ 88 eV fm^3

Branching in EC β Decay

ν phase space depends on
$Q = E_{max}$ →
rate λ increases with E_{max}

$$P_{if} = G_F^2 \frac{2 \cdot Z^3}{\pi^2 \hbar^4 c^3 a_B^3} E_\nu^2 \;; \quad E_\nu = E_{max} = Q$$

$$\lambda(E_{max}) \propto E_{max}^2$$



$$\frac{\lambda_{ex}(0.478\,MeV)}{\lambda_{gs}} = \frac{(Q - 0.478MeV)^2}{Q^2}$$

$$\frac{\lambda_{ex}}{\lambda_{gs}} = \left(\frac{0.382}{0.861}\right)^2 = 0.20$$

Experimental value correct magnitude but disagrees quantitatively

$$\left(\lambda_{ex}/\lambda_{gs}\right)_{exp} = 0.115$$

Reason: $\psi_n \neq \psi_p$ because of nuclear spin change $3^-/2$ → $1^-/2$
"forbidden" transition

Shape of the $\beta^\pm$ Spectrum

Beta decay other than EC
→ 3-body final state
Neglect nuclear recoil energy.

$$(N,Z) \rightarrow \begin{cases} (N-1, Z+1) + e^- + \bar{\nu}_e \\ (N+1, Z-1) + e^+ + \nu_e \end{cases}$$

$$\boxed{P_{if} = \frac{2\pi}{\hbar}\left|H_{fi}\right|^2 \cdot \rho(E_f)} \qquad \rho(E_f) = \frac{d(n_e \cdot n_\nu)}{dE_f} \qquad E_f = E_{max} = E_e + E_\nu$$

$$\frac{dn_\nu}{dp_\nu} = \frac{4\pi p_\nu^2}{h^3}V = \frac{4\pi V}{h^3}\frac{1}{c^2}\left(E_{\max} - E_e\right)^2 \qquad p_\nu \approx E_\nu/c \qquad \frac{dn_e}{dp_e} = \frac{4\pi p_e^2}{h^3}V$$

$\textit{plane waves for } e,\nu \rightarrow \left|H_{fi}\right|^2 \propto 1/V^2$ (problematic for $e^\pm$, Coulomb)

$\textit{Fixed } E_e \rightarrow dp_\nu/dE_{\max} = 1/c$

$$dn_e \cdot dn_\nu = \frac{V^2}{4\pi^4\hbar^6}\frac{1}{c^3}p_e^2 dp_e \cdot p_\nu^2 dp_\nu$$

$$dn_e \cdot \frac{dn_\nu}{dE_{\max}} = \frac{V^2}{4\pi^4\hbar^6 c^3}p_e^2 dp_e\left(E_{\max} - E_e\right)^2 = \rho(E_f)\,dp_e$$

$$\boxed{\frac{dN_e}{dp_e} = \frac{G_F^2\left|H_{fi}\right|^2}{2\pi^3\hbar^7 c^3} \cdot p_e^2 \cdot \left(E_{\max} - E_e\right)^2}$$

β momentum spectrum

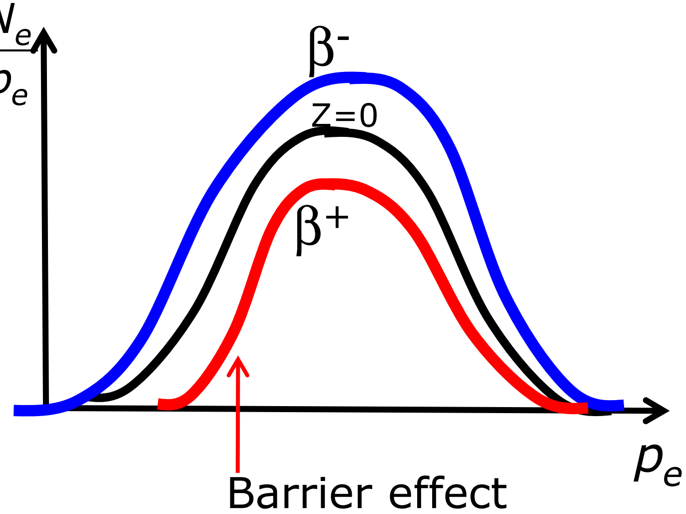
Shape of β± Spectrum/Coulomb Correction

Relativistic momentum-energy relation

$$E_e = W = \sqrt{(p_e c)^2 + (m_e c^2)^2} \qquad E_{max} = W_{max} \approx Q \quad (neglect\ nucl.\ recoil)$$

$$\frac{dW}{dp_e} = \frac{p_e c^2}{\sqrt{(p_e c)^2 + (m_e c^2)^2}} \qquad p_e c = \sqrt{W^2 - m_e^2 c^4}$$

$$\frac{dN_e}{dW} = \frac{G_F^2 |H_{fi}|^2}{2\pi^3 \hbar^7 c^5} \cdot p_e W \cdot (W_{max} - W)^2 = \frac{G_F^2 |H_{fi}|^2}{2\pi^3 \hbar^7 c^5} W \sqrt{W^2 - m_e^2 c^4}\,(W_{max} - W)^2$$



Should use **Coulomb** ψ_e (r) ≠ plane wave.
Electron cloud acts as barrier for e+. Nonrelativistic numerical correction factor (Fermi function)

$$F(Z, p_e) := |\psi_e(0)|^2 \Big/ |\psi_e^{free}(0)|^2 \approx \frac{2\pi\eta}{\{1 - \exp[-2\pi\eta]\}}$$

$$\eta := \pm \frac{e^2 Z}{\hbar \upsilon_e} \approx 2 \quad (for \quad \beta^{\mp})$$

$$\boxed{\frac{dN_e}{dp_e} = \frac{G_F^2 |H_{fi}|^2}{2\pi^3 \hbar^7 c^3} F(Z, p_e) p_e^2 (E_{max} - E_e)^2}$$

W. Udo Schröder, 2009

Total β± Decay Rate

Seek method to systematize data: Unit conversion

$$\tau_0 := \frac{2\pi^3\hbar^7}{m_e^5 c^4 G_F^2} \qquad \varepsilon := \frac{W}{m_e c^2} \qquad \pi := \frac{p_e}{m_e c}$$

$$\frac{dN_e}{d\varepsilon} = \frac{|H_{fi}|^2}{\tau_0}\varepsilon\sqrt{\varepsilon^2-1}\left(\varepsilon_{max}-\varepsilon\right)^2 \quad for\ F = 1,\ m_\nu = 0$$

$$\lambda = \int_1^{\varepsilon_{max}} d\varepsilon \frac{dN_e}{d\varepsilon} = \frac{\ell n2}{t_{1/2}}$$

Parameterization (Machner, 2005)

$$f(Z, E_{max}) = a(Z)\cdot E_{max}^{b(Z)}$$

$$a(Z) = \exp\{-5.553 + 7.3418\exp(Z/213.86)\}$$

$$b(Z) = 4.148\exp\{-Z/51.6\}$$

$Z > 0$ for β^-, $Z < 0$ for β^+

Coulomb Correction :

$$f(Z, \varepsilon_{max}) = \int_1^{\varepsilon_{max}} d\varepsilon\, F(Z,\varepsilon)\cdot\varepsilon\cdot\sqrt{\varepsilon^2-1}\cdot\left(\varepsilon_{max}-\varepsilon\right)^2$$

Universal numerical function, independent of spectrum → Tables

$$\lambda = \frac{|H_{fi}|^2}{\tau_0}\cdot f\left(Z, \varepsilon_{max}\right) = \frac{\ell n2}{t_{1/2}}$$

Nuclear structure information

$$|H_{fi}|^2 = G_F^2\left|\int_{Nucl} d^3\vec{r}\,\psi_f^*(\vec{r})\hat{\tau}_+\psi_i(\vec{r})\right|^2$$

Phase space : $f\left(Z, \varepsilon_{max}\right)$, τ_0

$\beta^{\pm}$ Decay *ft*-Values

Experimental task: E_{max}, and $t_{1/2}$ combination → nuclear matrix element

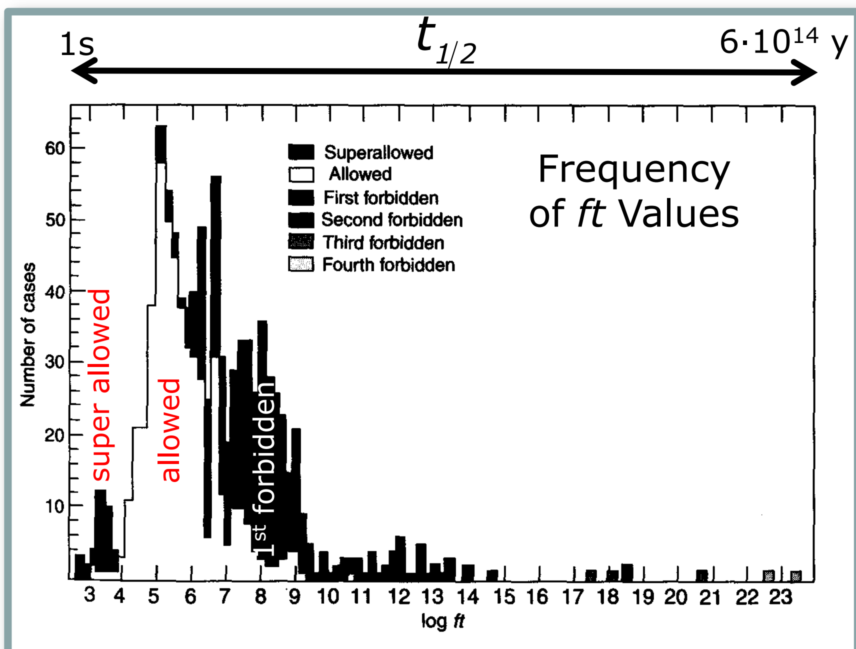
$$ft := f(Z, \varepsilon_{max})\, t_{1/2} = \frac{\tau_0 \cdot \ell n2}{|H_{fi}|^2}$$

$$B = \tau_0 \cdot \ell n2 = (2787 \pm 70)\, s$$

$$|H_{fi}|^2 = B/ft$$

$$ft := \frac{B}{|H_{fi}|^2}$$

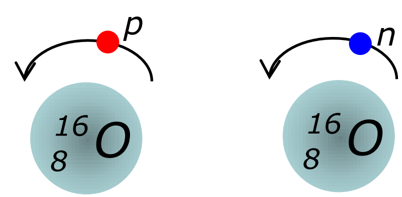
Large *ft*:
slow transitions, small $|H_{fi}|^2$



Meyerhof, 1967

"Super allowed" β transitions:
Large matrix elements, small *ft* observed only for light nuclei ("mirror nuclei") and $\Delta I = 0, \pm 1$

$${}^{17}_{7}F \xrightarrow{\beta^-} {}^{17}_{8}O \qquad \log ft = 3.38$$



"Allowed" β transitions: $\Delta I = 0, \pm 1$