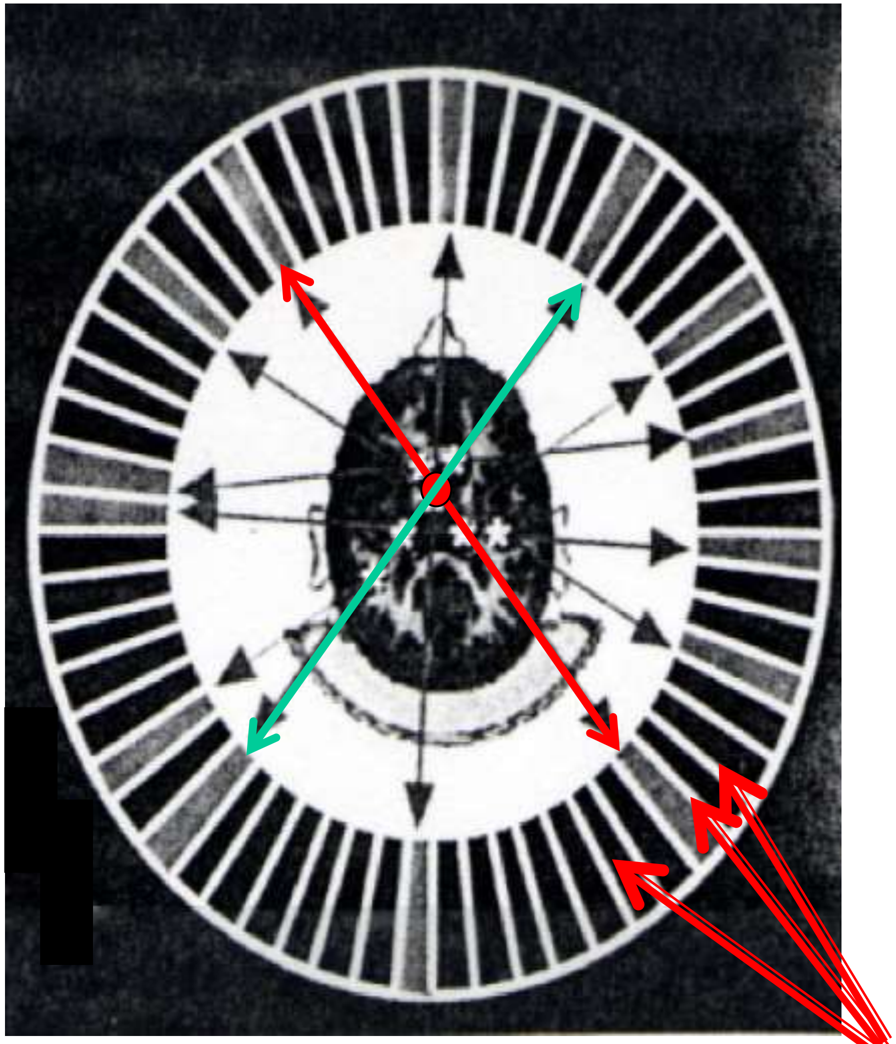
Today's Agenda

- PET Scan, γ–γ angular correlations
  - Coincidence measurements, electronics
  - Absolute activities
- Möβbauer Effect
  - Recoil effects in γ emission and absorption
  - Electronic setup
  - Applications: Electron-nuclear hyperfine interactions

Radiation Detectors for Medical Imaging

Positron emission tomographic (PET) virtual slice through patient's brain



Administer to patient labeled tracers, e.g.,
radioactive water: $H_2{}^{17}O$
radioactive acetate: $^{11}CH_3COOX$

Observe ^{17}O or ^{11}C β^+ decay mostly via β^+ annihilation

$$e^+ + e^- \rightarrow 2\gamma(511\ keV)$$

Positron e^+ (anti-matter) annihilates with electron e^- (its matter equivalent of the same mass) to produce pure energy (photons, γ-rays). Energy and momentum balance require back-to-back (180^0) emission of 2 γ-rays of equal energy

γ Detectors (e.g., NaI(*Tl*),…)

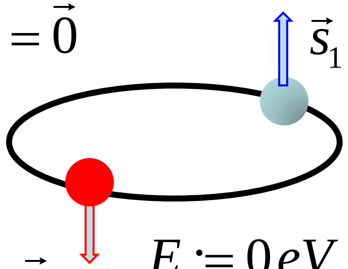
Positronium and e^+-e^- Annihilation

Decay at rest: $2E_\gamma \approx E_0 \approx 1.022\ MeV$

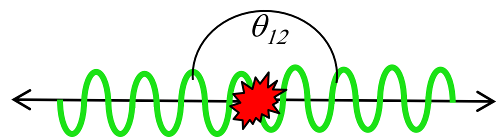
2-body decay → $\theta_{12} \approx 180^0$

Para Positronium

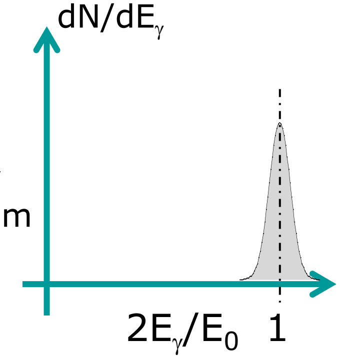
$\vec{I} = \vec{0}$



$E := 0\,eV$



2-body decay
→line spectrum
in E_γ and θ_{12}

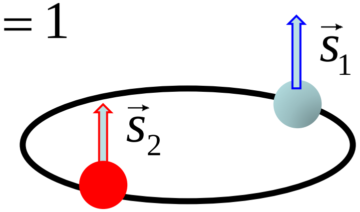


$$\tau_{2\gamma}(n) = 1.25 \cdot 10^{-10}\, n^3\ \text{sec}$$

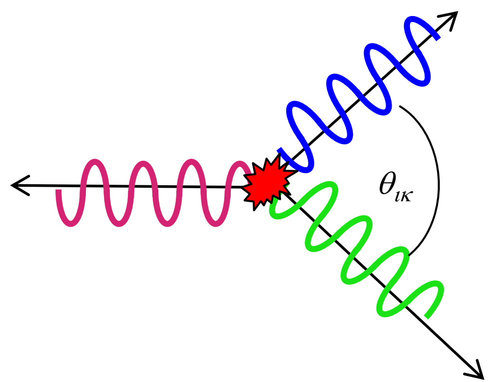
$$n = \textit{principal quantum \# of positronium molecule}$$

Ortho Positronium

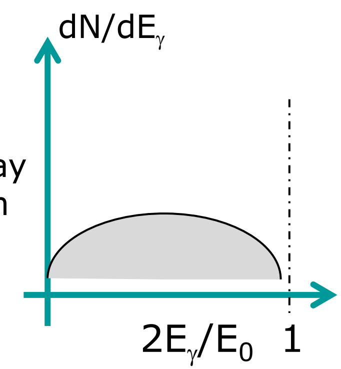
$\vec{I} = \vec{1}$



$E = 8 \cdot 10^{-4}\, eV$



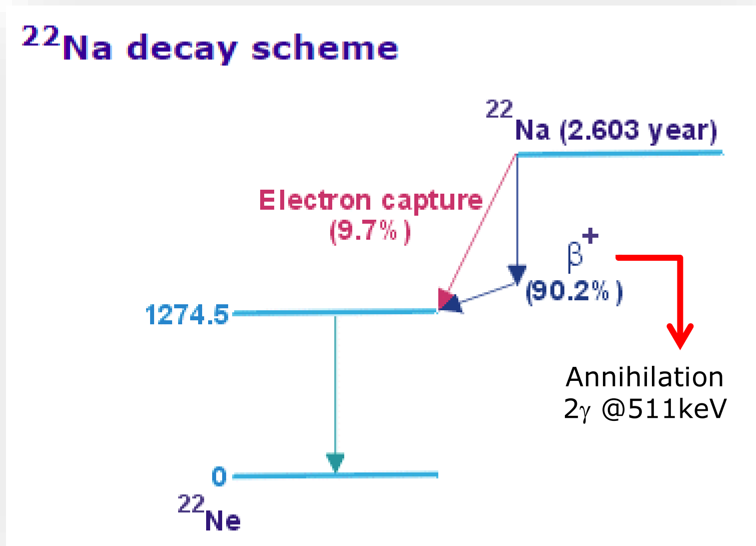
3-body decay
→continuum
in E_γ and $\theta_{\iota\kappa}$

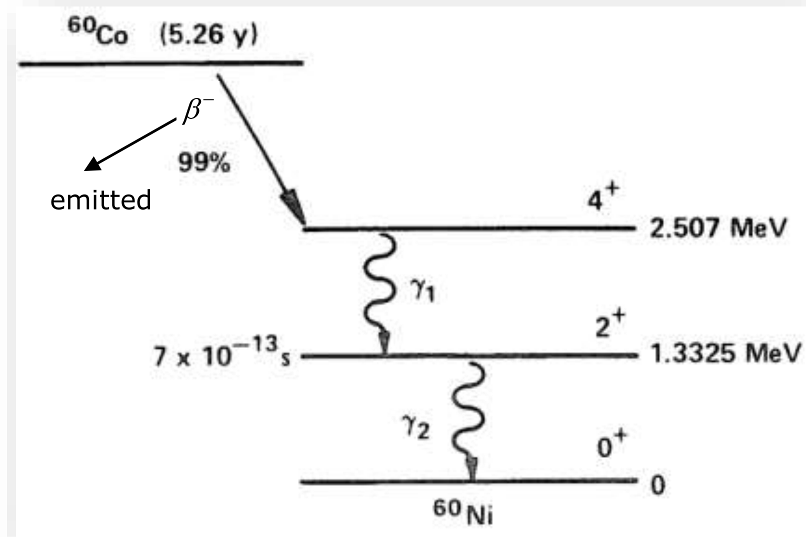


$\sigma_{2\gamma} = \pi r_0^2 \cdot \dfrac{\upsilon_{e^+e^-}}{c}$; $\quad \sigma_{2\gamma}/\sigma_{3\gamma} = 372$ $\quad \tau_{3\gamma}(n) = 1.4 \cdot 10^{-7}\, n^3\ \text{sec}$

$r_0 = 2.818\ fm$, *class. electron radius*

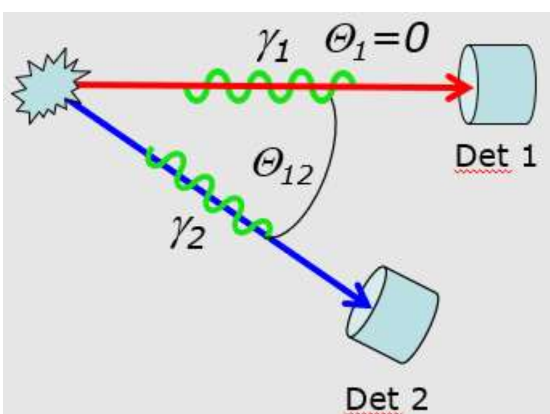
Gamma-Gamma-Correlations





Detection probabilities

$$P_i = \varepsilon_i \cdot \frac{\Delta\Omega_i}{4\pi}; \; \Delta\Omega_i = \frac{F_{detector\ i}}{R^2_{source-detector\ i}}$$



Absolute Activity Measurement

2 directionally uncorrelated γ-rays detected as singles (N_1 and N_2) or coincidences (N_{12}).

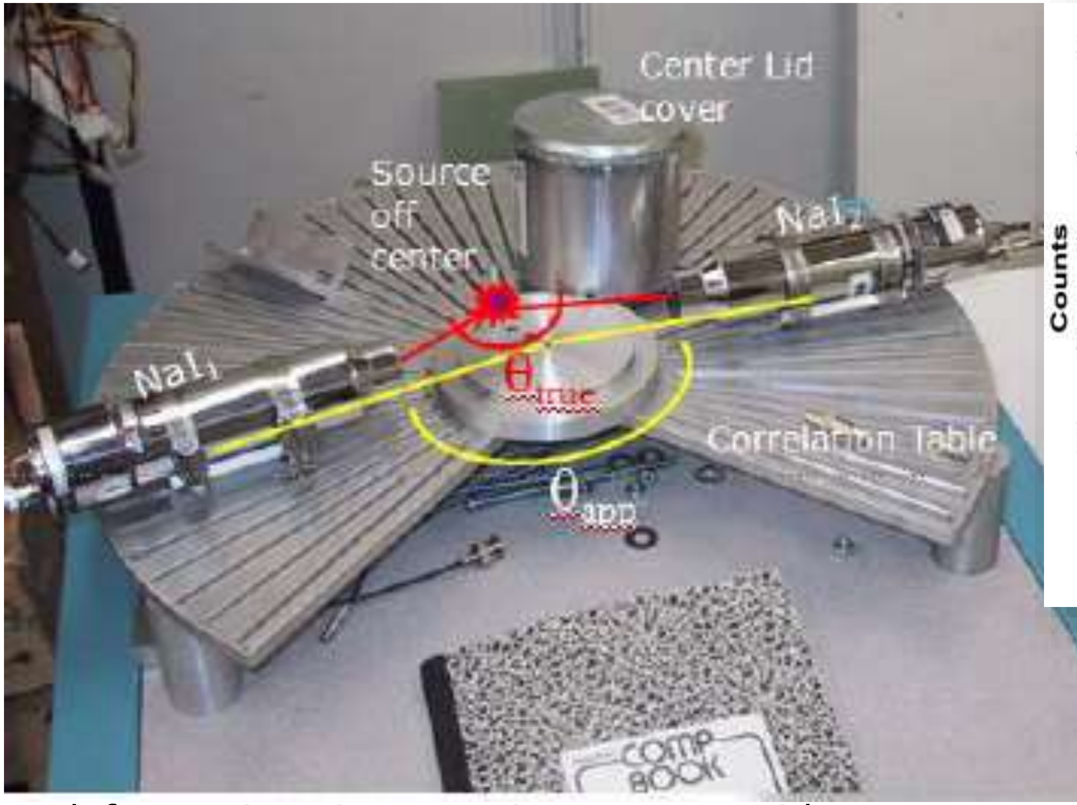
$N_1 = A \cdot P_1 \quad N_2 = A \cdot P_2$ *individual rates*

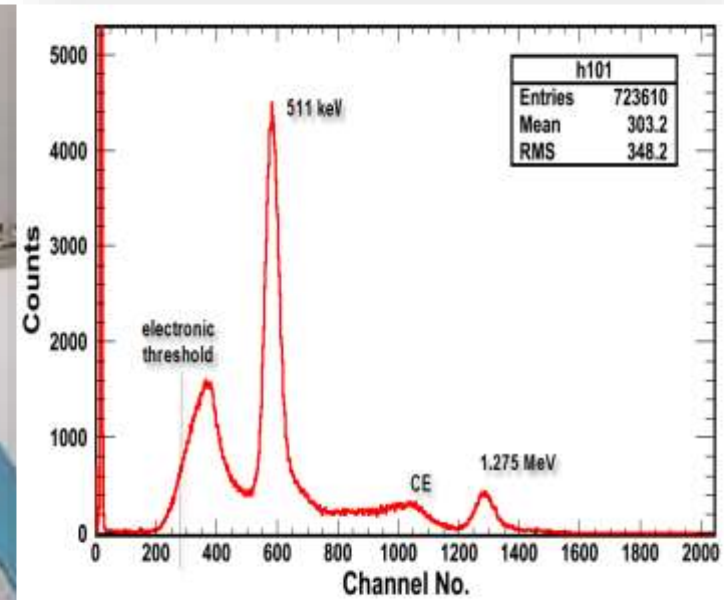
$P_{12} = P_1 \bullet P_2 \quad N_{12} = A \cdot P_{12}$ *uncorr. coinc rate*

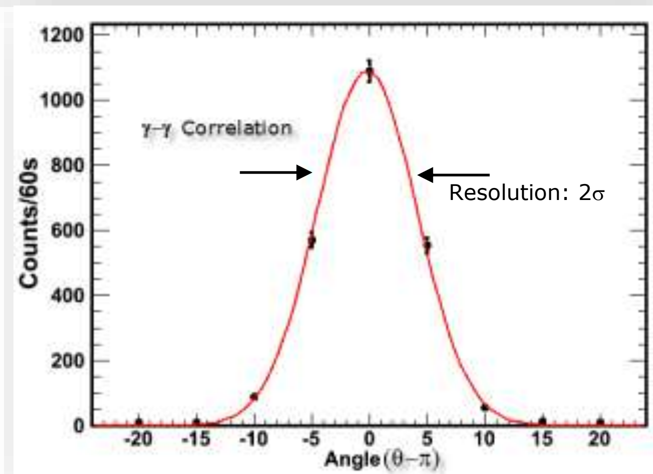
$$A = \frac{N_1 \cdot N_2}{N_{12}} = \frac{A \cdot P_1 \cdot A \cdot P_2}{A \cdot P_{12}}$$ *singles / coinc*

A error ??

ANSEL Angular Correlation Experiment





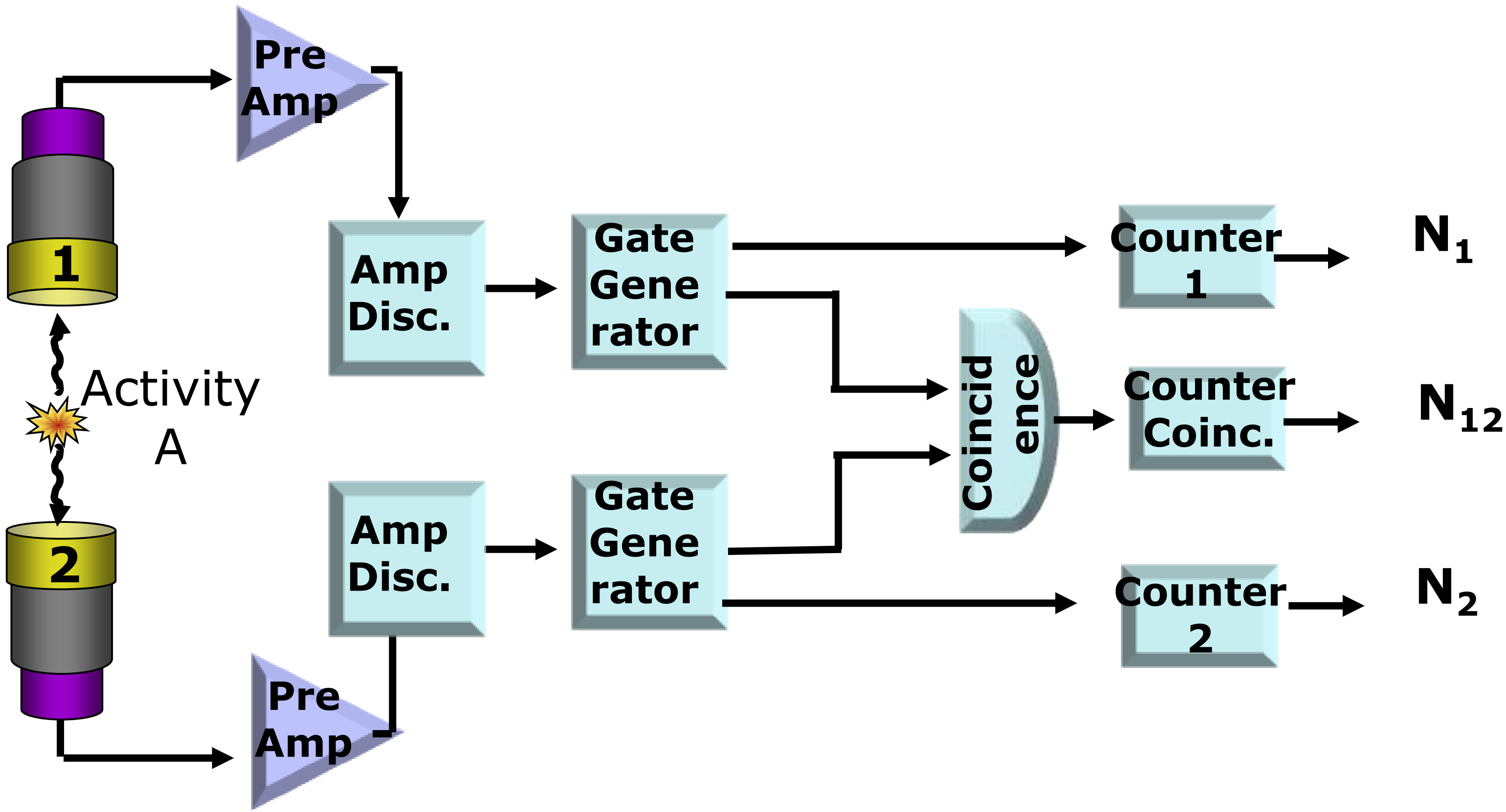


Top left: PET imaging experiment setup with two 1.5″x1.5″ NaI(*Tl*) detectors (BICRON) on a slotted correlation table. A "point-like" ^{22}Na γ source can be hidden from view.
Top right: ^{22}Na γ spectrum measured with NaI_1.
Bottom right: $\gamma-\gamma$ angular correlation measurement.

Second correlation setup: NaI*(Tl) vs. HP*Ge

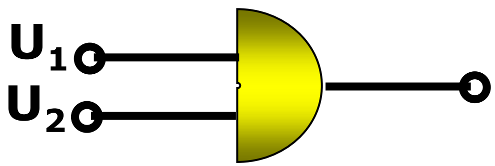
E_γ-Ungated Coincidence Measurement

Activity $A=\lambda N$ [disintegrations/time], simultaneous emission of (angular-) independent radiation types: ***i =1,2*** in event, resp. detection probabilities P_i

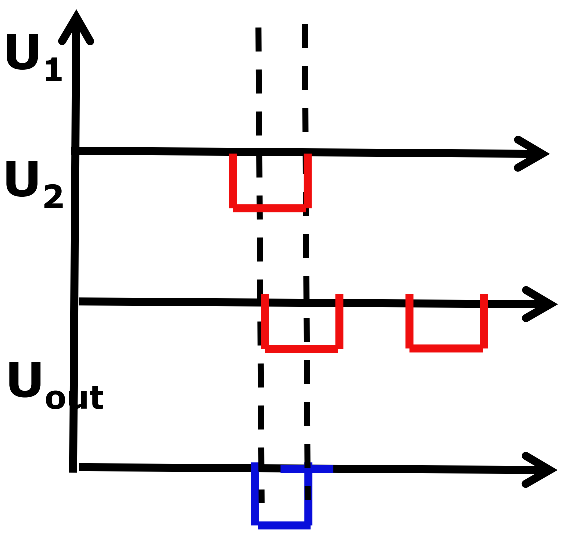


Digital Logic Modules

AND Overlap Coincidence

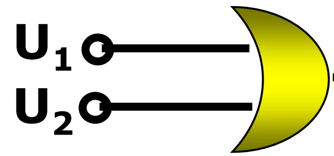


$U_{out} = U_1 \wedge U_2$

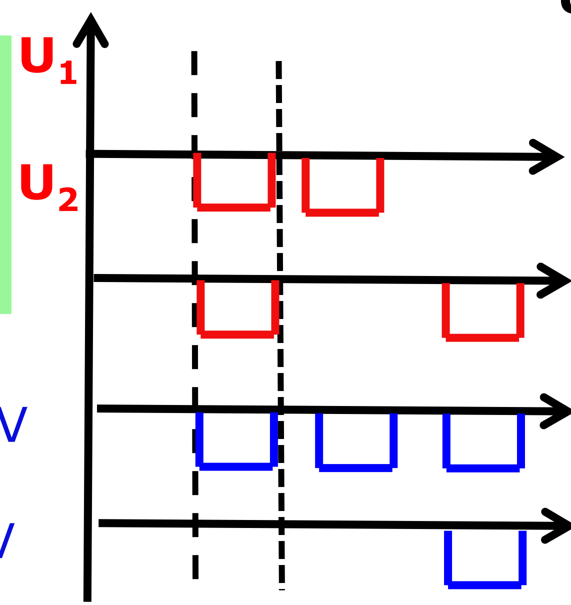


For fast timing: use fast negative logic

OR (inclusive)



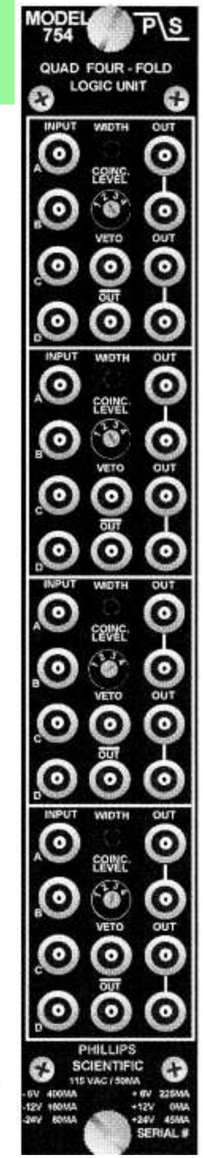
$U_{out} = U_1 \vee U_2$



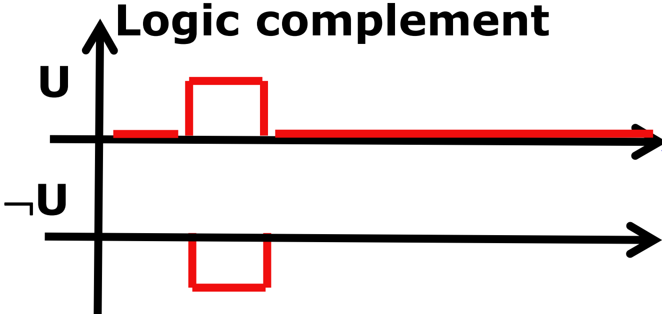
inc $U_1 \vee U_2$

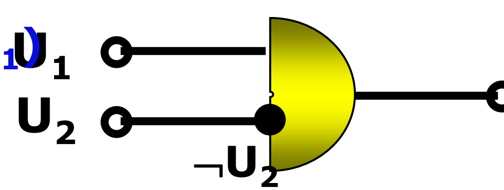
ex $(U_1 \vee U_2) \wedge \neg (U_2 \wedge U_1)$

Quad 4-fold majority logic unit



Logic complement





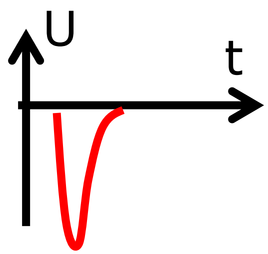
$U_{out} = U_1 \wedge \neg U_2$

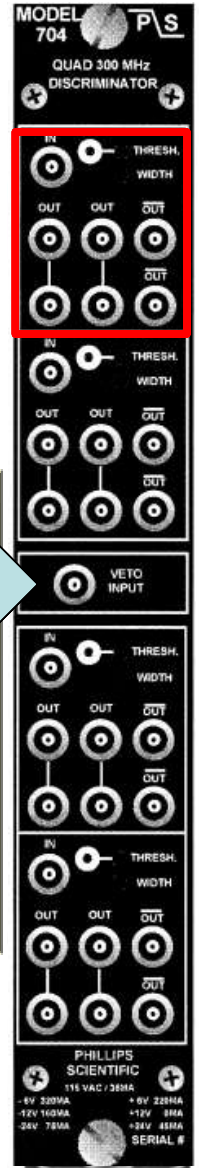
Anti-Coincidence

Logic Chain Elements: Fast NIM Modules

Fast LE Discriminator

Pre/Amp





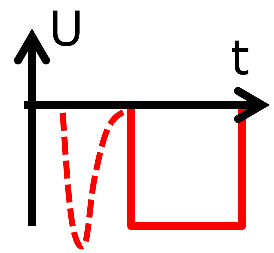
Input: fast,
narrow NIM sig

Veto

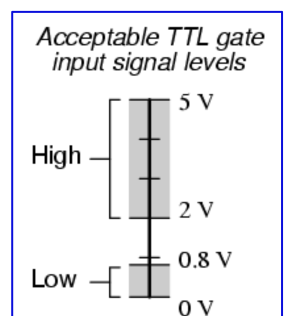
NIM = current based logic, with negative “true”
I = -16 mA
@ 50 Ω → - 0.8 V

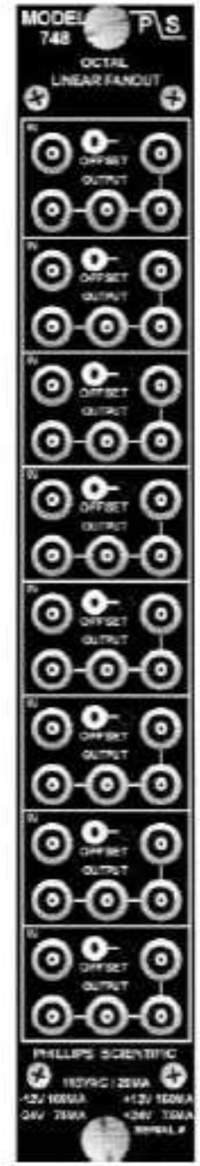
Check on 50 Ω termination

Gate & Delay Generator



Output: Long NIM/TTL “gate’ signal

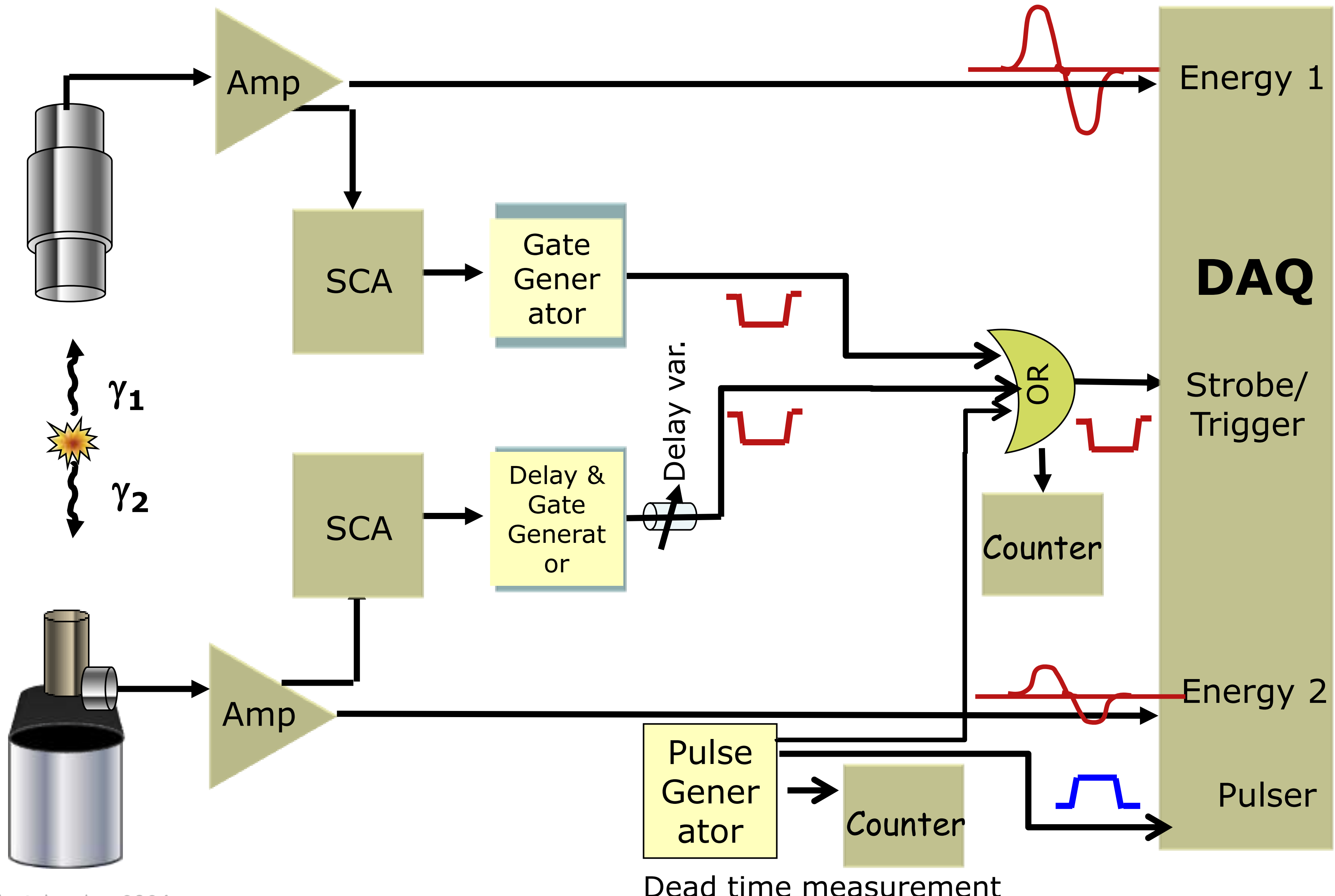




Fan In/ Fan Out Module

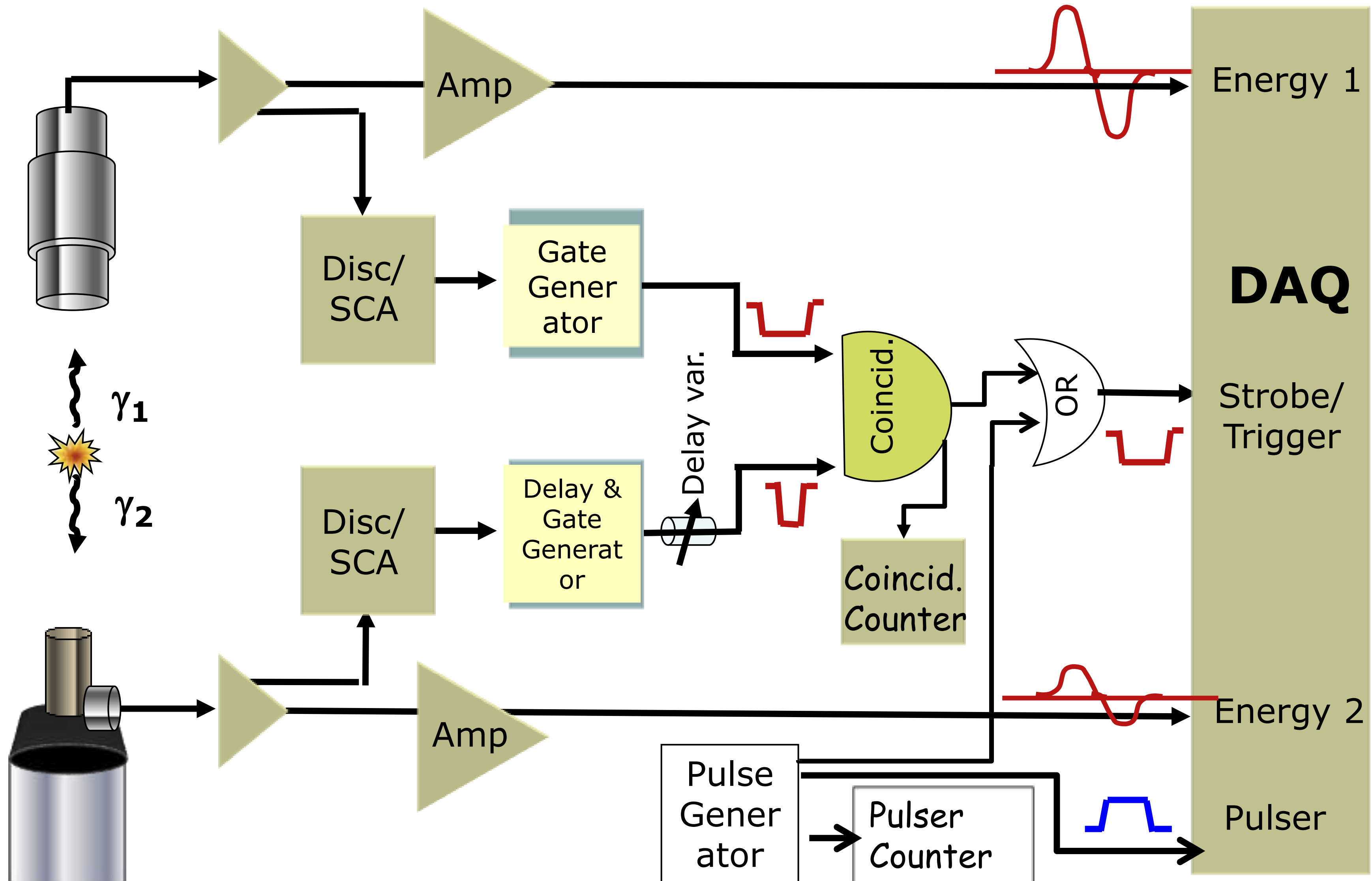
Gated E_1, E_2 Inclusive Measurement

PET (511keV **V** 511keV) or γ–γ cascade (^{60}Ni), gates on γ_1 and γ_2 lines



Gated E_1-E_2 Coincidence Measurement

PET (511keV $\wedge$ 511keV) or γ–γ cascade (^{60}Ni), gates on γ_1 and γ_2 lines



Continuous dead time measurement

Digital Data Structure

Example of event stream with signals (observables) measured simultaneously in 3 inputs of the Data Acquisition Module (DDC-8), trigger signal: inclusive OR (det1 V det2 V det3)
Sample below displays 6 successive events, 3 of them are "coincidences."

Single event "wave 0"
Single event "wave 1"
Single event "wave 2"

Event#	Input 0 Channel #	Input 1 Channel #	Input 2 Channel #
51	1542	0	0
52	1530	0	3
53	1486	0	0
54	1789	256	0
55	1547	0	0
56	1533	0	0

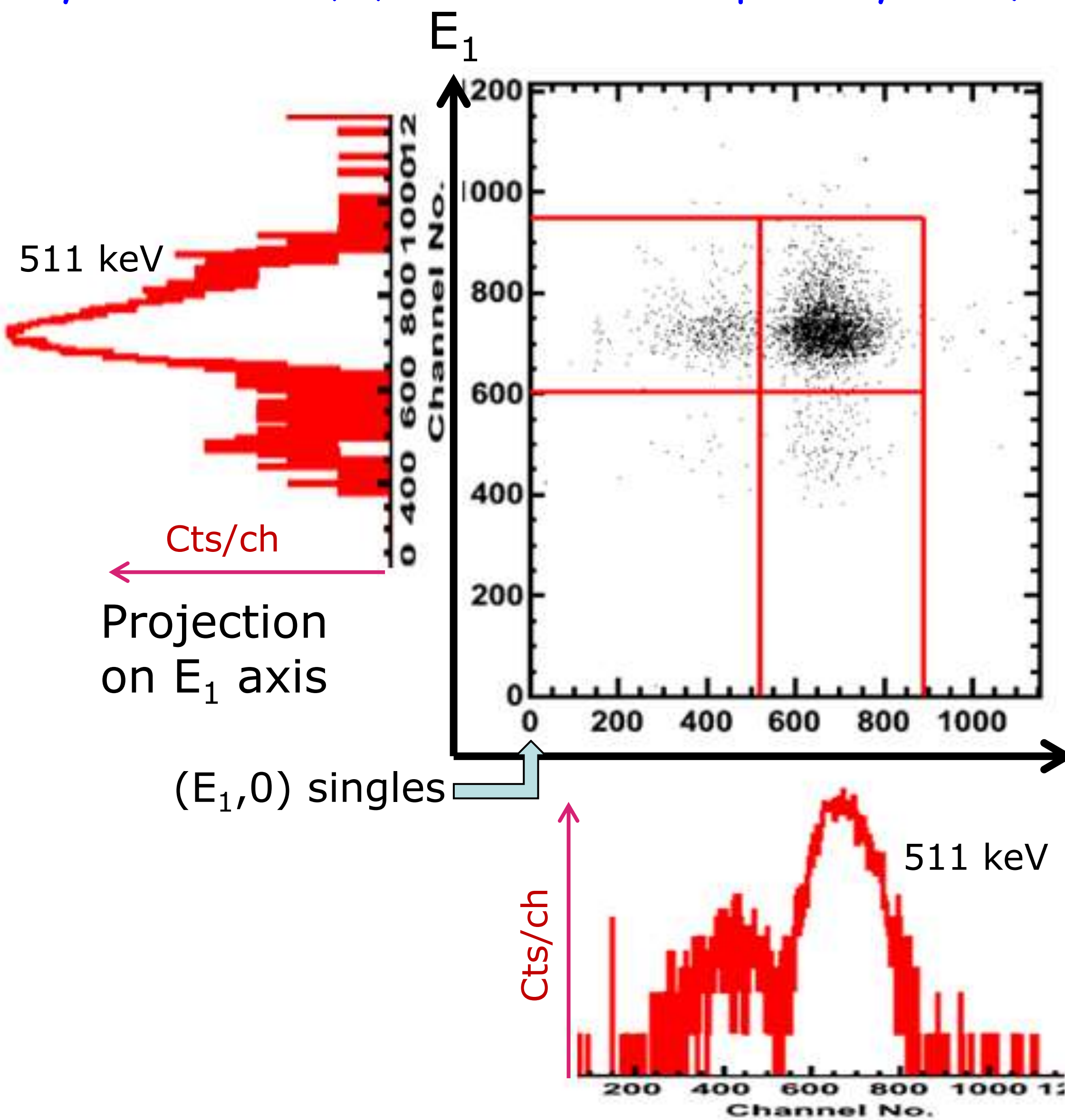
Coincident event 0 .AND. 2 (event 52)

Coincident event 0 .AND. 1 (event 54)

With OR trigger, coincidence "resolution" is given by slow DAQ electronics.
Fast front-end determination reduces random background.

2D Parameter Coincidence Measurement

Only coincident γ–γ events are accepted by DAQ.



2D Scatter Plot:

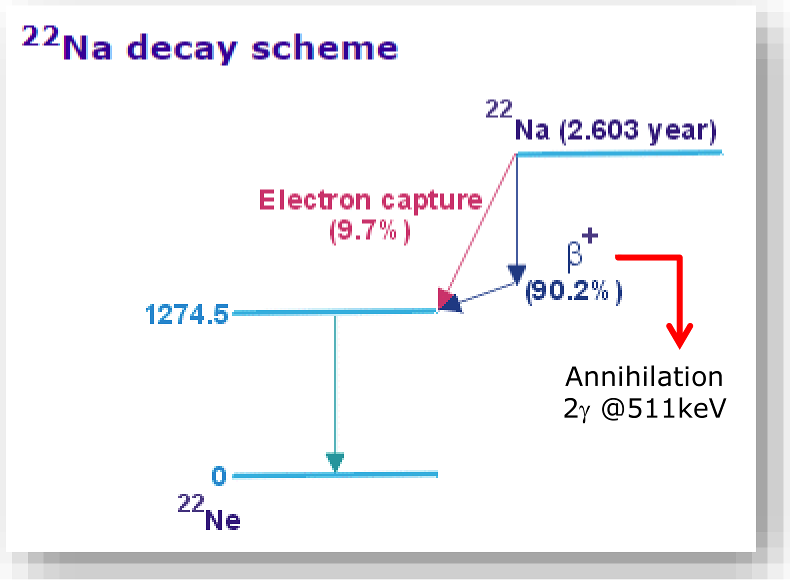
Each point represents one 2D event $\{E_1, E_2\}$

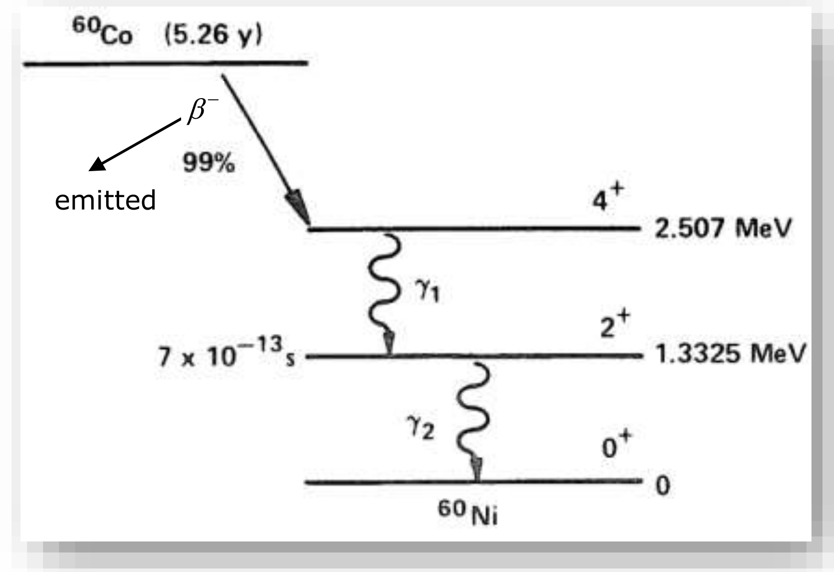
"Minimum Bias" → Multiplicity=2

$(0, E_2)$ singles

Projection on E_2 axis

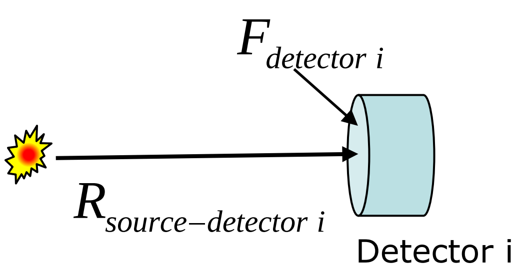
Gamma-Gamma-Correlations





Detection probabilities

$$P_i = \varepsilon_i \cdot \frac{\Delta\Omega_i}{4\pi};\ \Delta\Omega_i = \frac{F_{detector\ i}}{R^2_{source-detector\ i}}$$



Absolute Activity Measurement

2 directionally uncorrelated γ-rays detected as singles (N_1 and N_2) or coincidences (N_{12}).

$N_1 = A \cdot P_1 \qquad N_2 = A \cdot P_2 \qquad$ *individual rates*

$\underline{P_{12} = P_1 \bullet P_2} \qquad N_{12} = A \cdot P_{12} \qquad$ *uncorr. coinc rate*

$$A = \frac{N_1 \cdot N_2}{N_{12}} = \frac{A \cdot \cancel{P_1} \cdot A \cdot \cancel{P_2}}{A \cdot \cancel{P_{12}}}$$

singles / coinc

A error ??

Today's Agenda

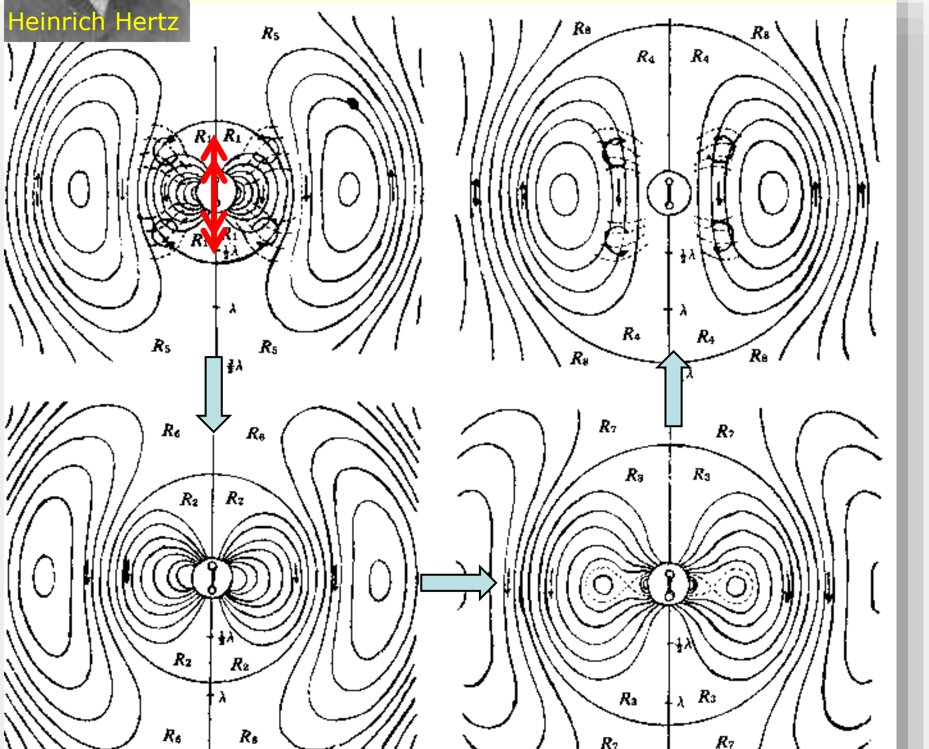
- Coincidence measurements, electronics PET Scan
  - Absolute activities
- $\gamma - \gamma$ angular correlations in nuclear deexcitation cascades
- Mößbauer Effect
  - Recoil effects in γ emission and absorption
  - Electronic setup
  - Applications: Electron-nuclear hyperfine interactions

Patterns of Propagating Electromagnetic Radiation Fields

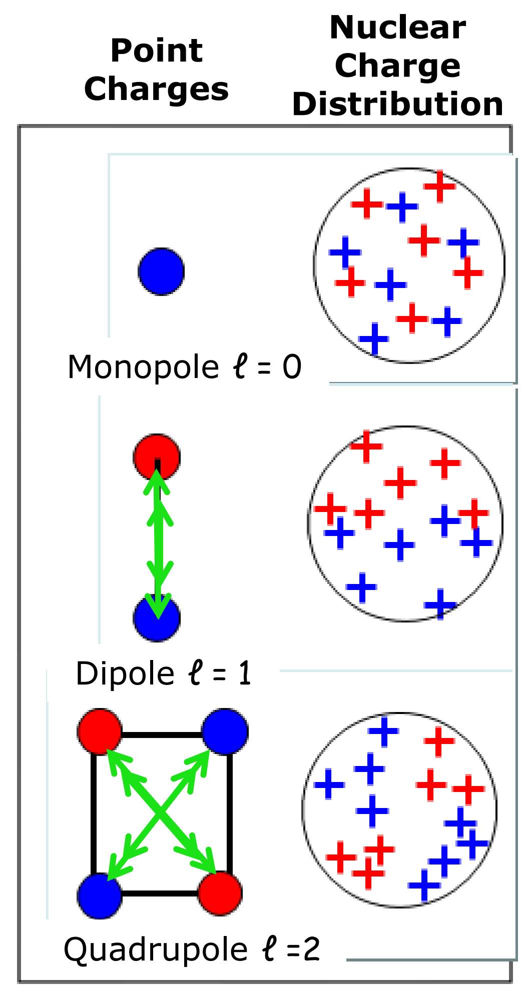
Anisotropic nuclear charge distribution. Changes → elm. rad.

Different charge *multipoles* → different spatial symmetries

Propagating *Electric Dipole*
Field Lines



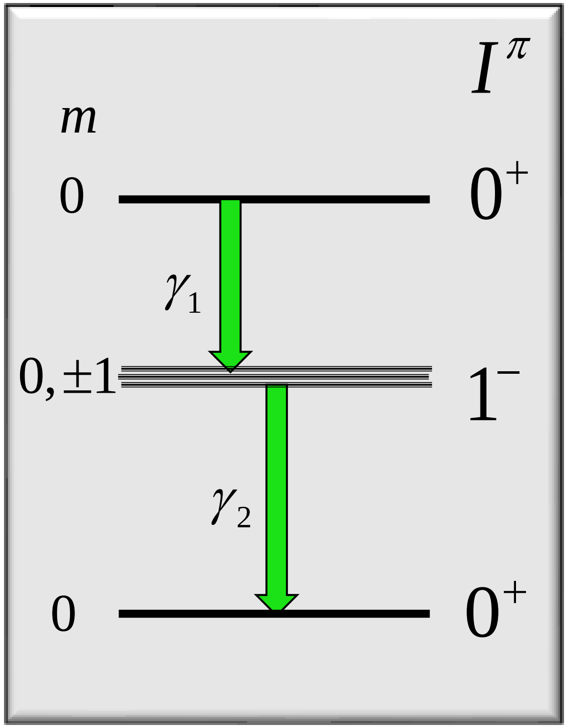
Elm field amplitude $\psi_{\Delta E}(r,t)\cdot Y_{\mu}^{\ell}(\theta,\varphi)$



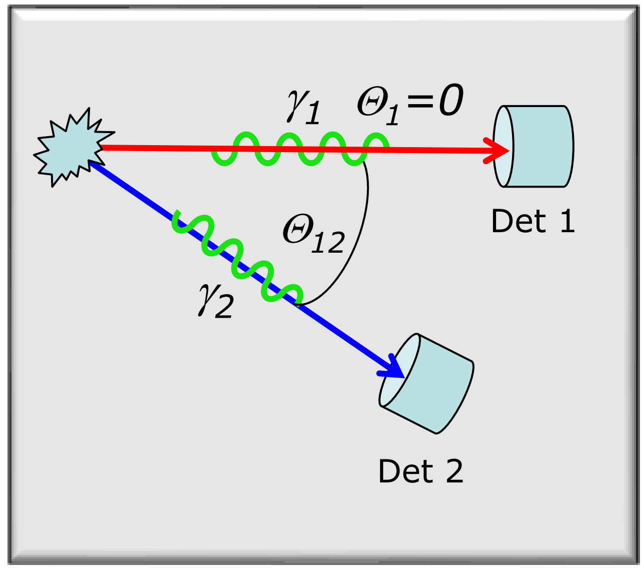
E. Segré: *Nuclei and Particles, Benjamin&Cummins, 2nd ed. 1977*

γ–γ Angular Correlations

Quantum mechanics of angular-momentum coupling of radiation to nuclear Ψ



Simple example: γ cascade 0 → 1 → 0
Mostly $\Delta m=\pm 1$ emitted in z-direction



Det 1: Define z, select transition with maximum γ intensity for $\theta_1=0$.

Det 2: Measure emission patterns with respect to this z direction → determine Δm.

$$W_{\gamma_1\gamma_2}(\theta) = W^{\gamma_1\gamma_2}_{\Delta m=\pm1}(0)\cdot W^{\gamma_2}_{\Delta m=\mp1}(\theta)$$
$$\propto W^{\gamma_2}_{\Delta m=\mp1}(\theta) \propto \left|Y_1^1(\theta)\right|^2$$
$$\propto \left(1+\frac{1}{2}\cos^2\theta\right)$$

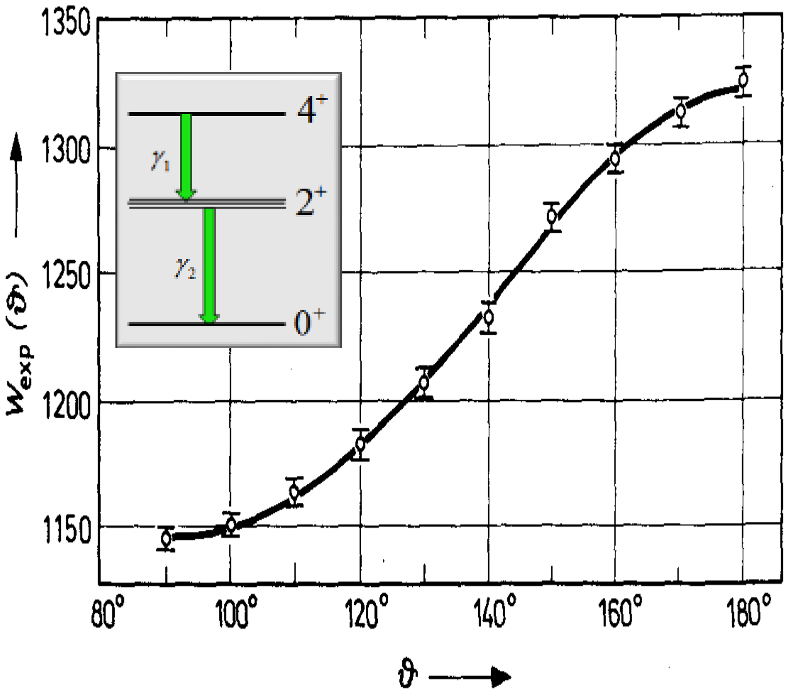
General quantal expression for γ–γ angular correlation

$$W_{\gamma_1\gamma_2}(\theta) = \sum_{n=0}^{\ell} A_{2n}P_{2n}\left(\cos\theta\right) \propto \sum_{n=0}^{\ell} A'_{2n}\cos^{2n}\theta$$
$$\propto 1 + A'_2\cos^2\theta + \ldots.$$

Typically: n ≤ 2. Legendre Polynomials $P_n(\cos\theta)$

E2 γ–γ Angular Correlations

Example: Rotational E2-γ cascade,
$\Delta m = \pm 2$ maximally emitted in z-direction



Theoretical $4^+ \xrightarrow{E2} 2^+ \xrightarrow{E2} 0^+$

$\ell = 2 \rightarrow$ highest order P_4

$$W(\theta) = 1 + 0.1020 \cdot P_2(\cos\theta) + 0.0091 \cdot P_4(\cos\theta)$$

$90^0/180^0$ Anisotropy

$$A_{\gamma\gamma} := \frac{W(90^0) - W(180^0)}{W(180^0)} = \sum_{n=1}^{n_{max}} A'_{2n}$$

After: G. Musiol, J. Ranft, R. Reif, D. Seeliger, *Kern-u. Elementarteilchenphysik*, VCH 1988

Experimental task: determine $\boldsymbol{A}_{\gamma\gamma}$ for the Co-60 γγ cascade.