Detection Of Ionizing Radiation

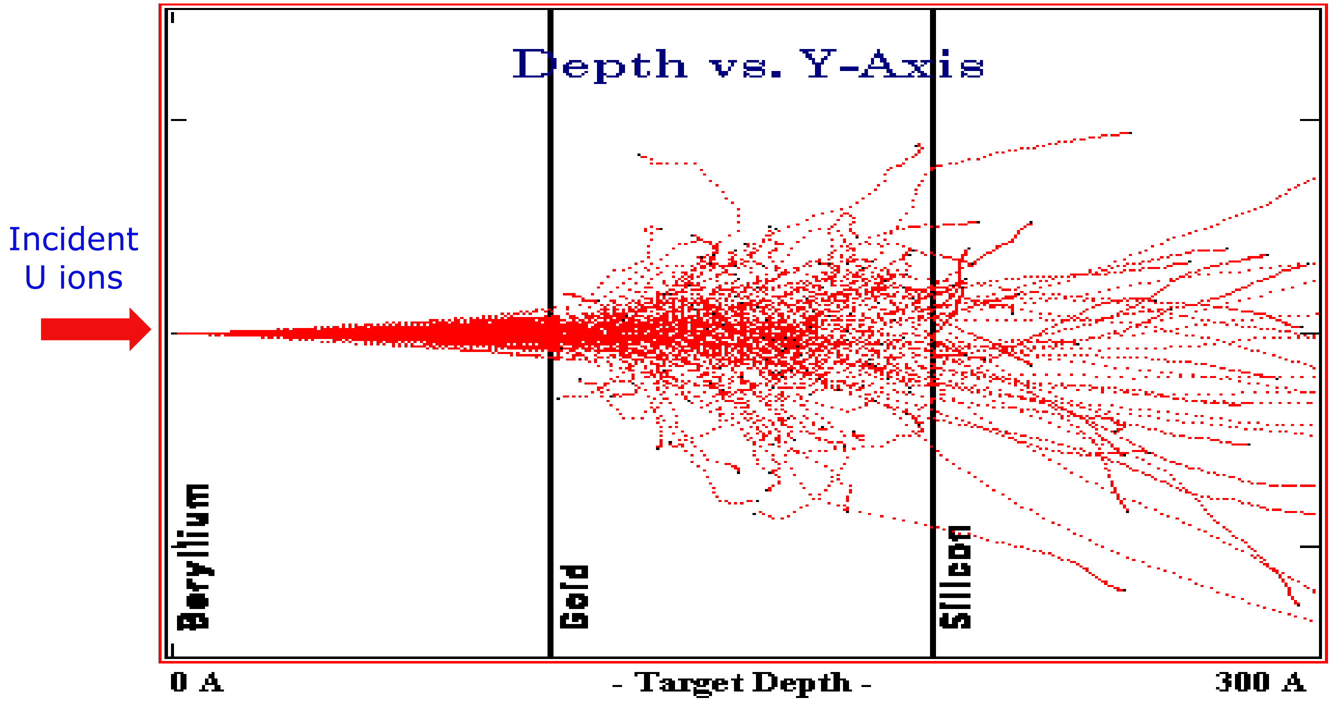
Charged Particles

Bethe-Bloch Theory

Ionization Mechanisms

- Gamma-rays: Photo-effect, Compton Effect, pair production.
- Charged particles: Coulomb interactions with absorber/target electrons. (G.F. Knoll, Ch.2 I&II)

→ Electrons as main free charge carriers



Main Interactions of Charged Particles

Dominant type of interaction: collisions with atomic electrons ⇒
- Atomic excitation
- ionization
- fluorescence
- phosphorescence

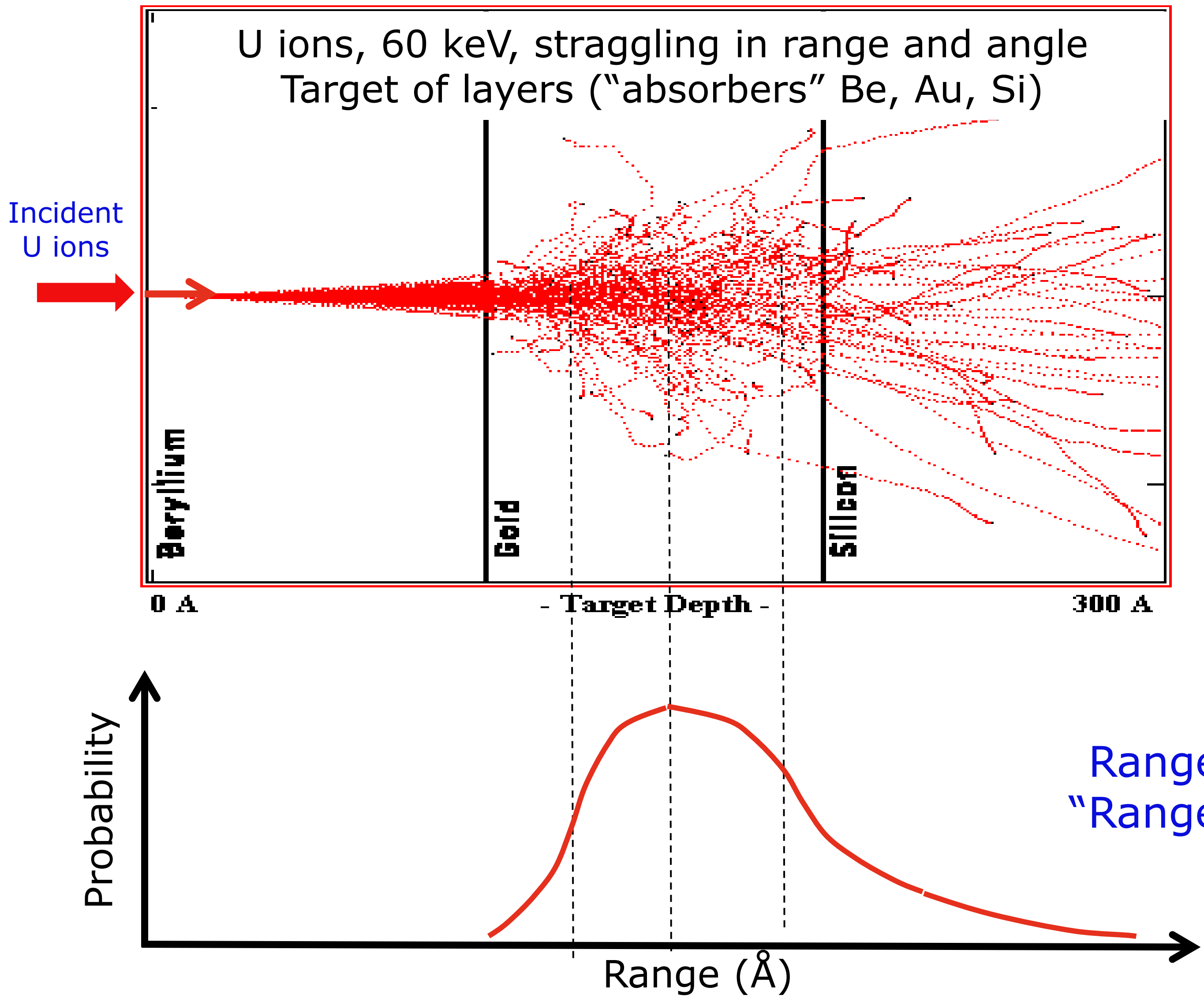
Probability for collisions with nuclei :

$$\frac{\sigma_{nucl}}{\sigma_{atom}} \sim \frac{\pi R_{nucl}^2}{\pi a_Z^2} \sim \frac{Z^2 \cdot 10^{-26} cm^2}{10^{-16} cm^2} \sim Z^2 \cdot 10^{-10} < 10^{-6}$$

Most interactions of charged particles with material components *are collisions with atomic electrons*.

Nuclear collisions are noticeable only at very low particle kinetic energies.

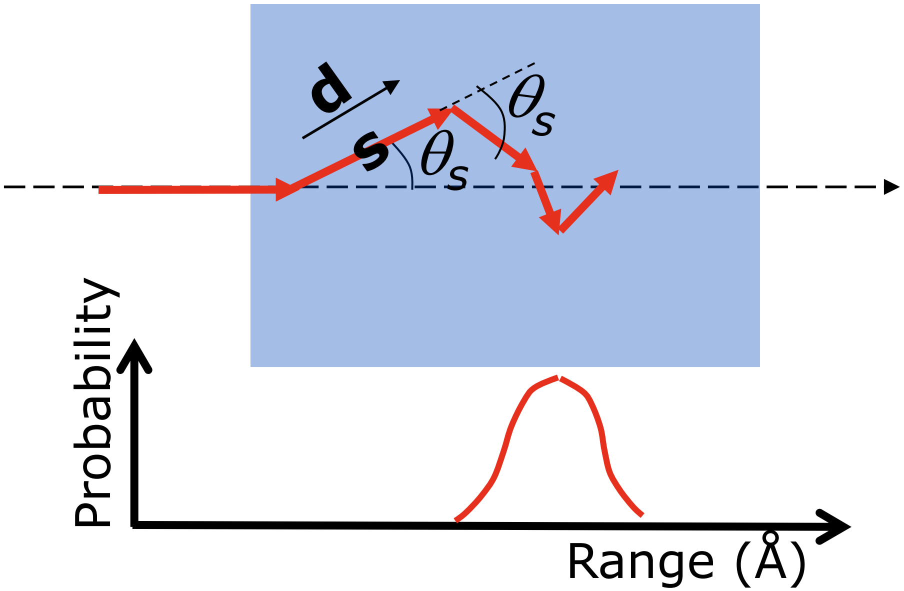
Stochastic Multiple Scattering and Straggling



Ziegler range program

Range is diffuse:
“Range straggling”

Range and Stopping Power



Scattering angle θ_s path variable s

Stochastic multiple scattering process produces straggling in range, energy loss, angle →

Particles traversing an absorber attain broader energy spectrum

$$R(E) = \int ds \langle \cos\theta_s \rangle = \int_0^E dE' \left[\frac{dE'}{ds}\right]^{-1} \cdot \langle \cos\theta_s \rangle$$ Range

$$\left[\frac{dE}{ds}\right]$$ Stopping power

$$S(E) = \int ds \geq R(E)$$ Path length of trajectory

Alpha Particle Range: Heuristic Formulas

Range in air (ρ=1.293 g/cm^3)

$$R_{Air}(cm) = \begin{cases} 0.56 \cdot E_\alpha(MeV) & for\ E_\alpha \le 4MeV \\ 1.24 \cdot E_\alpha(MeV) - 2.62 & 4MeV < E_\alpha \le 8MeV \end{cases}$$

Range in other absorbers (atomic number =A)

$$R_\alpha(E_\alpha, A) = 0.56 \cdot A^{1/3} \cdot R_{Air}(E_\alpha)\ mg/cm^2$$

Example: Range of ^{210}Po alphas E_α= 5.3 MeV in Al (ρ=2.7 g/cm^3)

$$R_{Air}(cm) = 1.24 \cdot 5.3 - 2.62 = 3.95$$

$$R_\alpha(5.3, 27) = 0.56 \cdot 27^{1/3} \cdot 3.95\, mg/cm^2 = 6.64\, mg/cm^2$$

$$R_\alpha(5.3, 27) = \frac{6.64\, mg/cm^2}{\rho_{Al}} = \frac{6.64 \cdot 10^{-3}\, g/cm^2}{2.7\, g/cm^3} = 246\, \mu m$$

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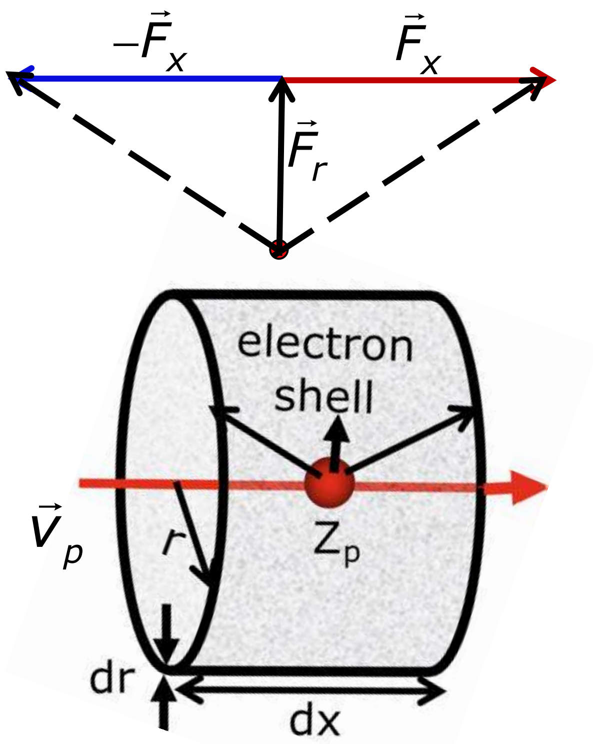
Phenomenological Model Of Energy Loss in Matter

Bethe et al. (1930-1953), Lindhardt's electron theory describes energy loss through ionization, incoming ions are fully stripped

Estimate of trends/Order of magnitude E=particle kinetic energy,

e^- ≈ at rest in lab system; but in cms $v_e \approx -v_p$

Lateral force cancellation



$$\Delta p_e = F_{Coul} \cdot \Delta t_{coll} \approx \left(k_0 \cdot \frac{e^2 Z_p}{r^2} \right) \cdot \left(\frac{2r}{v} \right)$$

$$k_0 = 1/4\pi\varepsilon_0 = 8.99 \cdot 10^9 \, Nm^2 C^{-2}$$

$$\text{electron density } \rho_e \left[m^{-3} \right]$$

$$-dE(r,x) \sim \left[2\pi r \, dr \, dx \, \rho_e \right] \cdot \left[\frac{(\Delta p_e)^2}{2m_e} \right]$$

$$-\frac{dE(r,x)}{dr\,dx} \sim \left[\frac{4\pi e^4 Z_p^2}{m_e v^2} \rho_e \right] \cdot \frac{1}{r}$$

Mass of e^-

$m_e c^2 = 0.511 MeV$

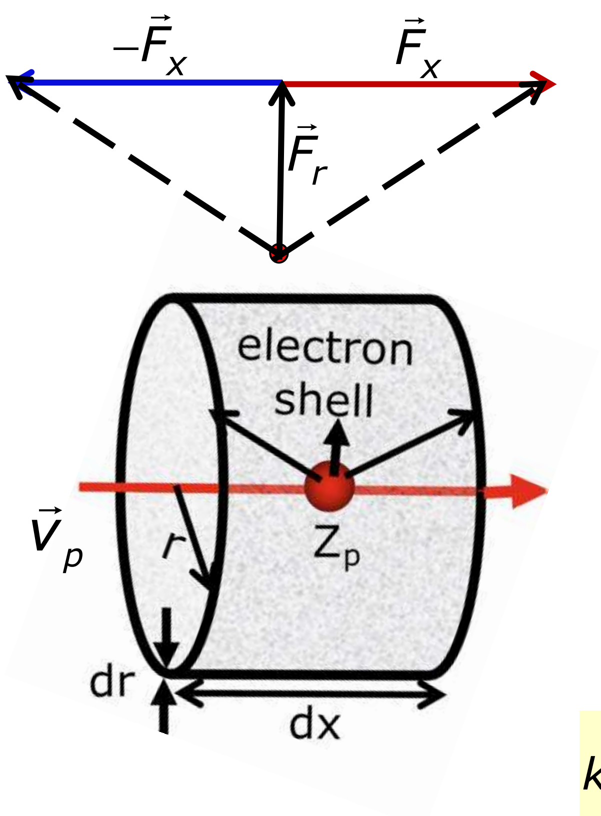
Important are only forces ⊥ to trajectory, others cancel

Phenomenological Model Of Energy Loss in Matter

Important are forces ⊥ to trajectory → Integrate over radial coordinate:

$$\frac{dE(x)}{dx} = \int_{r_{min}}^{r_{max}} dr \frac{dE(r,x)}{dr\,dx} \quad \Rightarrow \quad \boxed{\frac{-dE(x)}{dx} \sim \left[k_0^2 \frac{4\pi e^4 Z_p^2}{m_e v_p^2} \rho_e \right] \cdot \ln\left(\frac{r_{max}}{r_{min}} \right)}$$

Lateral force cancellation



Estimate of radial limits: ρ_e = e⁻ density

λ_e = electronic wavelength, cms $|v_e| \approx |v_p|$

$$r_{min} \gtrsim 2 \cdot \lambda_e;\ \lambda_e = h/p_e = h\sqrt{1-\beta_p^2}/m_e v_p$$

non – adiabatic motion (fast particle, low Δt_{coll})

$$\Delta t_{coll} \le \frac{1}{\langle \nu_e \rangle} \rightarrow \frac{r_{max}}{v_p} = \frac{T_e}{\sqrt{1-\beta_p^2}} = \frac{h}{\left[IE \cdot \sqrt{1-\beta_p^2} \right]}$$

T_e = average e^- orbital period → IE/h = frequency

IE = ionization energy

$$k_0^2 \frac{4\pi e^4}{m_e c^2} \cdot \frac{1}{m^3} = 8.12 \cdot 10^{-42} \frac{J}{m} = 5.08 \cdot 10^{-31} \frac{MeV}{cm}$$

Electronic Stopping Power: Bethe-Bloch Formula

$$\left(\frac{-dE}{dx}\right)_p \approx \left(\frac{k_0^2 4\pi e^4}{m_e c^2}\right)\left(\frac{Z_p}{\beta_p}\right)^2 \cdot \rho_{Abs}^{el} \cdot \left(Ln\frac{2m_e c^2 \beta_p^2}{I_E\left(1-\beta_p^2\right)} - \beta_p^2 + \ldots\right)$$

Next order (→ β_p^2)

$$k_0^2 \frac{4\pi e^4}{m_e c^2} = \frac{5.08\cdot 10^{-25}}{MeV\cdot cm^2}$$

$$m_e c^2 = 0.511 MeV$$

Specify absorber material : density, volume, atomic number Z_{abs}; *mass number* A_{abs} = # *of nucleons (n, p) per particle, Number of electrons per particle* $n_{el} = Z_{abs}$

→ *Density of electrons in absorber* $= \rho_{Abs}^{el} \uparrow$

"*Molecular weight*" $W_{mol} = A_{abs} \cdot 1g$ *contains* $L = 6.02\cdot 10^{23}$ *particles*

Volume $= V_{abs}\left[m^3\right]$, *Density* $= \rho_{abs}\left[g/m^3\right]$

$$\rightarrow \rho_{Abs}^{el} = L\cdot\frac{W_{abs}}{W_{mol}}\cdot\frac{n_{el}}{V_{abs}} = L\cdot\frac{\rho_{abs}}{W_{mol}}\cdot n_{el} \rightarrow \rho_{Abs}^{el} \sim \frac{Z_{abs}}{A_{abs}}$$

Contains absorber isotopic information

Ionization energy

$$IE = h\langle\nu_e\rangle \approx \begin{cases}(12\cdot Z_T + 7)eV & for\ Z_T < 13\\ (9.76\cdot Z_T + 58.8\cdot Z_T^{-0.19})eV & for\ Z_T \ge 13\end{cases}$$

$$For\ a\ compound\ \left\{rel.density\ \rho_i = \frac{n_{el,i}}{n_{el}}, Z_i\right\} \rightarrow LnI_E = \sum_i \rho_i \cdot Z_i \cdot LnI_{E,i}$$

$$Water: I_{E,H} = 19eV,\ I_{E,O} = 105eV \rightarrow I_{E,H_2O} = 75eV$$

The Bethe-Bloch Formula

$$\left(\frac{-dE}{dx}\right)_p \approx 5.08\cdot 10^{-25}\,MeVcm^2\cdot\rho^{el}_{Abs}\cdot\frac{Z_p^2}{\beta_p^2}\cdot\left(Ln\frac{2m_ec^2\beta_p^2}{I_E\left(1-\beta_p^2\right)}-\beta_p^2\right)$$

IE= Ionization energy

$$Absorber\ Examples:\ hydrogen\ gas\ A_{abs}=1,\ n_{el}=1$$

$$Water\ {}^1_1H_2{}^{16}_8O\rightarrow A_{abs}=2\times 1+16=18;\quad W_{mol}=18g;\ n_{el}=10$$

$$\rho^{\ell}_{abs}=10^6\,g/m^3\rightarrow\rho_{el}=L\cdot\left(\rho^{\ell}_{abs}/W_{mol}\right)\cdot n_{el}=3.34\cdot 10^{29}m^{-3}=3.34\cdot 10^{23}cm^{-3}$$

$$Protons\ in\ liquid\ H_2O:\ \frac{-dE}{dx}\approx\frac{0.1697}{\beta^2}\cdot\left(Ln\frac{2m_ec^2\beta^2}{\left(1-\beta^2\right)}-\beta^2-LnI_E\right)\frac{MeV}{cm}$$

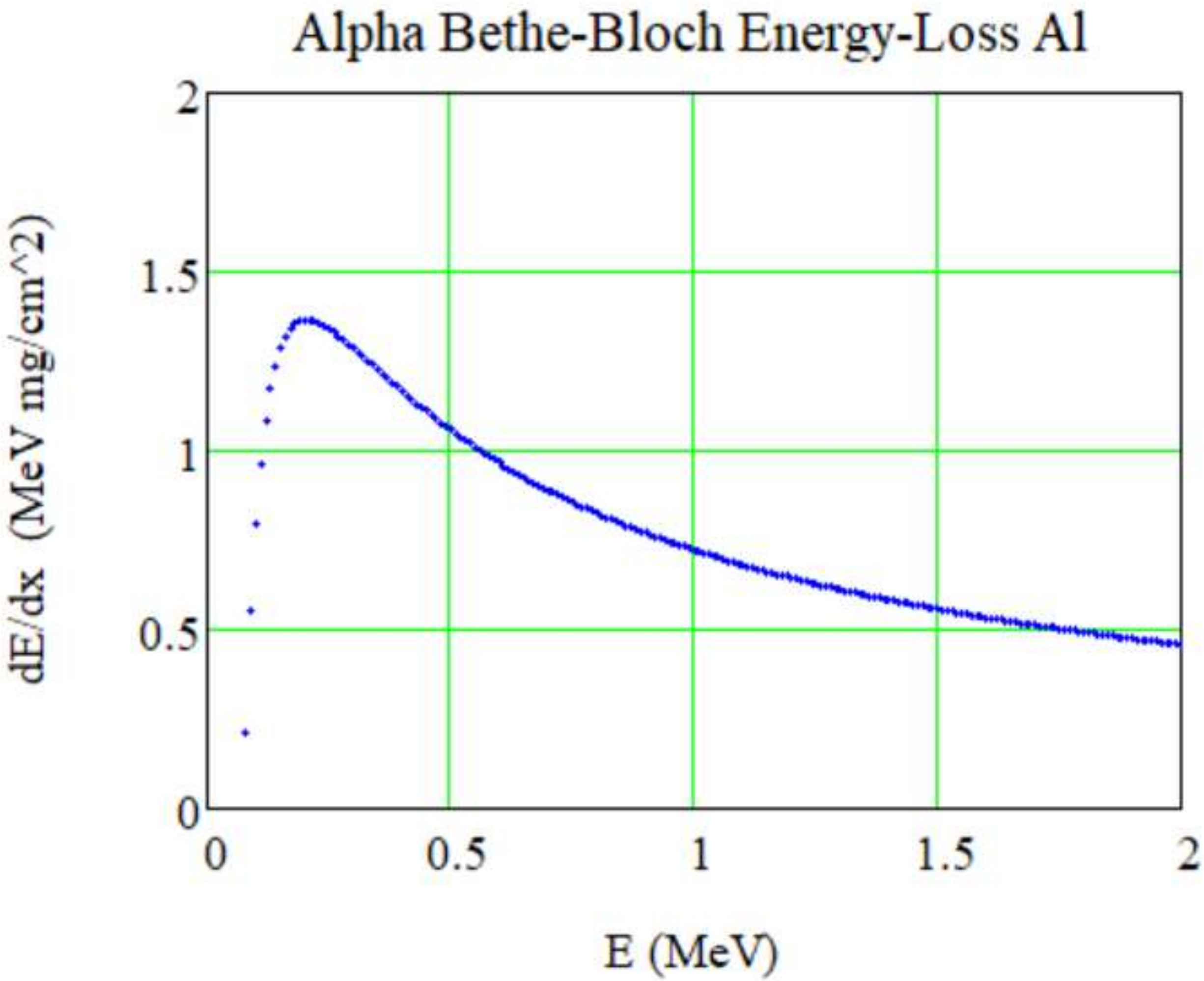
$$Example:\ Proton\ kinetic\ energy\ E=1MeV\rightarrow\beta^2=0.0021$$

$$\frac{-dE}{dx}\approx\frac{0.1697}{\beta^2}\cdot\left(Ln\frac{0.0021\cdot 1.022\cdot 10^6eV}{75eV\cdot 0.9979}-0.0021\right)\frac{MeV}{cm}=270\frac{MeV}{cm}=2.70\frac{keV}{\mu m}$$

In popular absorber units (g/cm^2)

$$\left(\frac{-dE}{d\left(\rho_{Abs}\cdot x\right)}\right)_p\approx 0.3071\left(\frac{Z}{\beta}\right)_p^2\cdot\left(\frac{Z}{A}\right)_{Abs}\cdot\left(Ln\frac{2m_ec^2\beta^2}{I_E\left(1-\beta^2\right)}-\beta^2\right)\frac{MeV}{g/cm^2}$$

Theoretical E-Loss in Thin Absorbers



^{210}Po alphas

E_α= 5.3 MeV

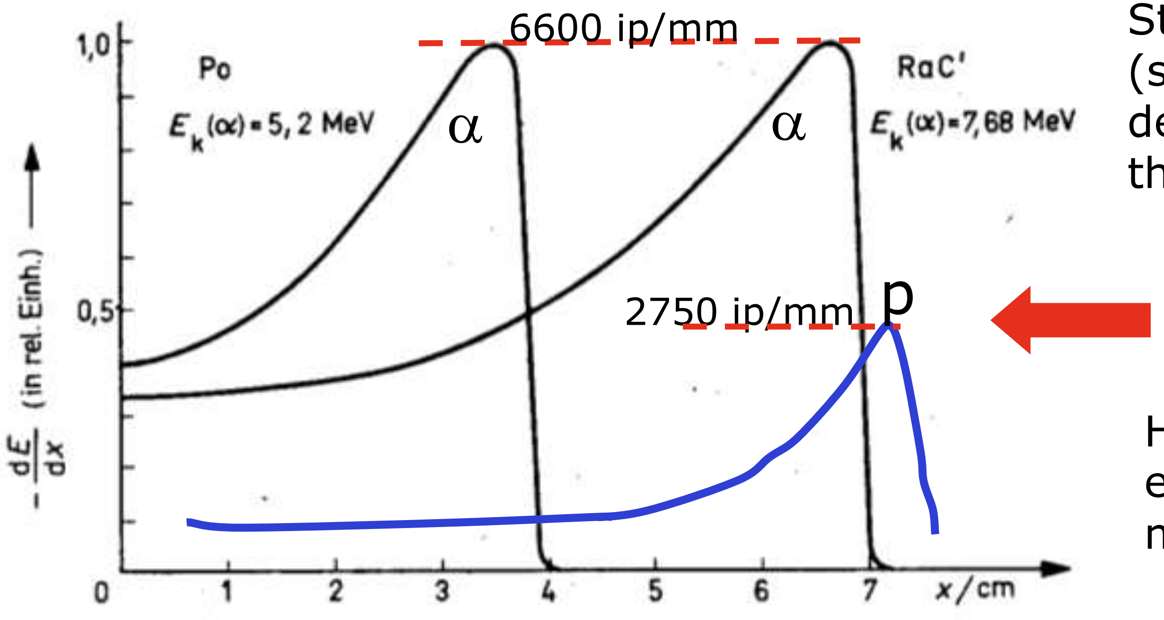
Loss:
MeV per mg/cm^2

Bragg maximum

Semi-quantitative only: Expt values smaller both at small and large energies → recharging effects for projectile

Range and Specific Ionization

E-loss in Air: 1atm, 15^0C



Stopping power dE/dx (specific energy loss) depends on energy E and therefore on x

Bragg Curve

Highest E loss close to end of path → Bragg maximum

$$\frac{dE}{dx} \triangleq \frac{\#\left(e^- - ion\ pairs\right)}{unit\ \ length}$$

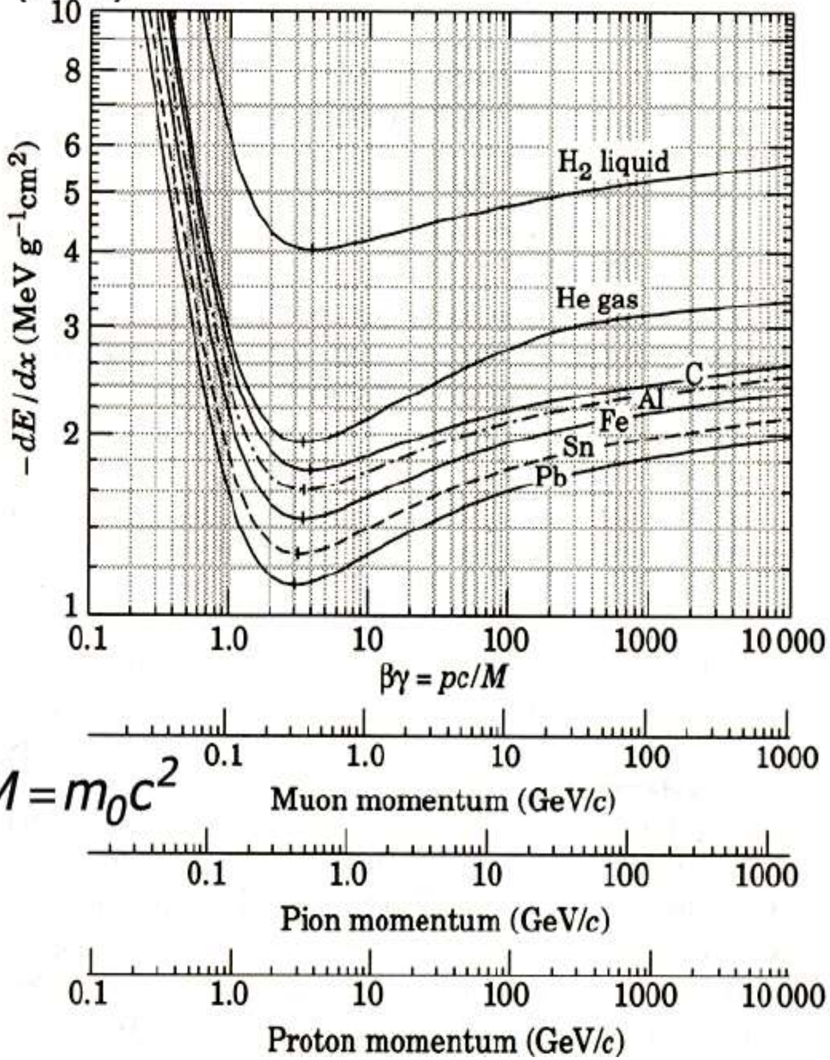
α *particles :*

$$\frac{dE}{dx} \leq \frac{7 \cdot 10^3\ pairs}{mm}$$

Main E-loss mechanism: ionization, production of "δ **electrons**," electron-ion pairs

Stopping Power Curves

Review of Particle Properties, Phys. Rev. D50, 1173 (1994)



$$M = m_0 c^2$$

$$E^2 = (pc)^2 + \left(m_0 c^2\right)^2$$

$$m_0 = particle\ rest\ mass$$

$$v = particle\ velocity$$

$$c = 2.9979 \cdot 10^8 ms^{-1}$$

$$\beta = v/c$$

$$\gamma = \left(\sqrt{1-\beta^2}\right)^{-1}$$

$$pc = (\gamma m_0) vc = \gamma\beta m_0 c^2$$

"mips"
=minimum ionizing particles

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