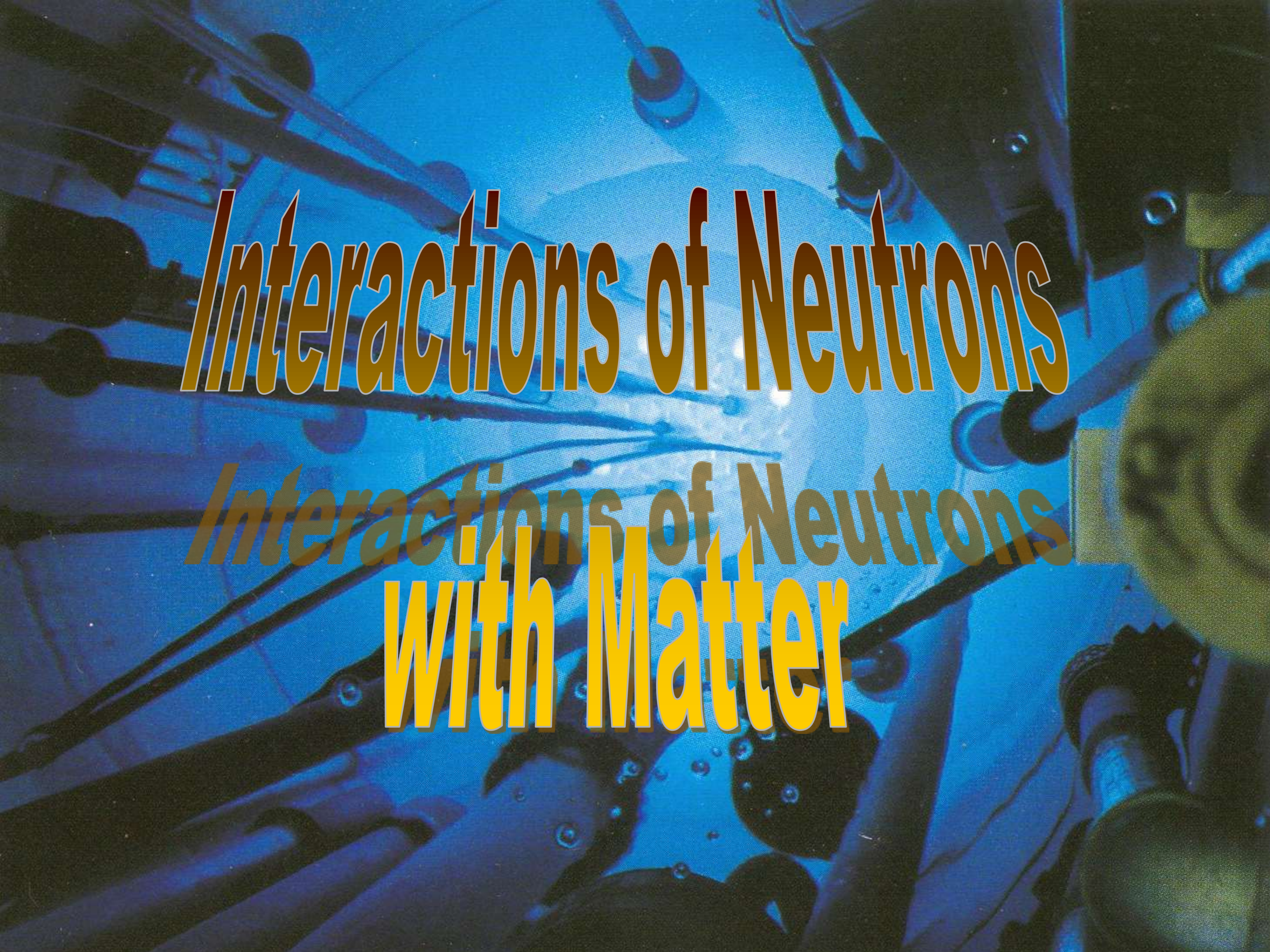
# Interactions of Neutrons with Matter



# Nuclear Interactions of Neutrons

No electric charge → no direct atomic ionization → only collisions and reactions with nuclei → $10^{-6}$ x weaker absorption than charged particles

Processes depend on available n energy $E_n$:

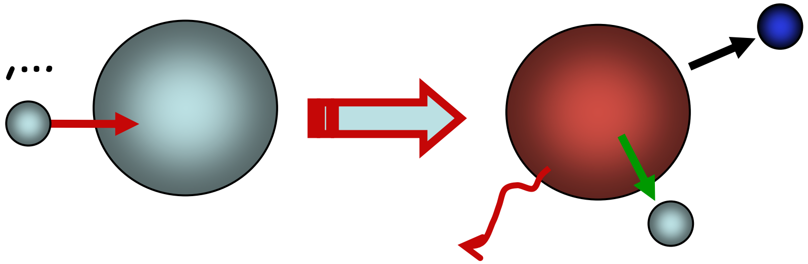
$E_n \sim 1/40$ eV (= $k_B T$) Slow diffusion, capture by nuclei

$E_n$ < 10 MeV Elastic scattering, capture, nucl. excitation

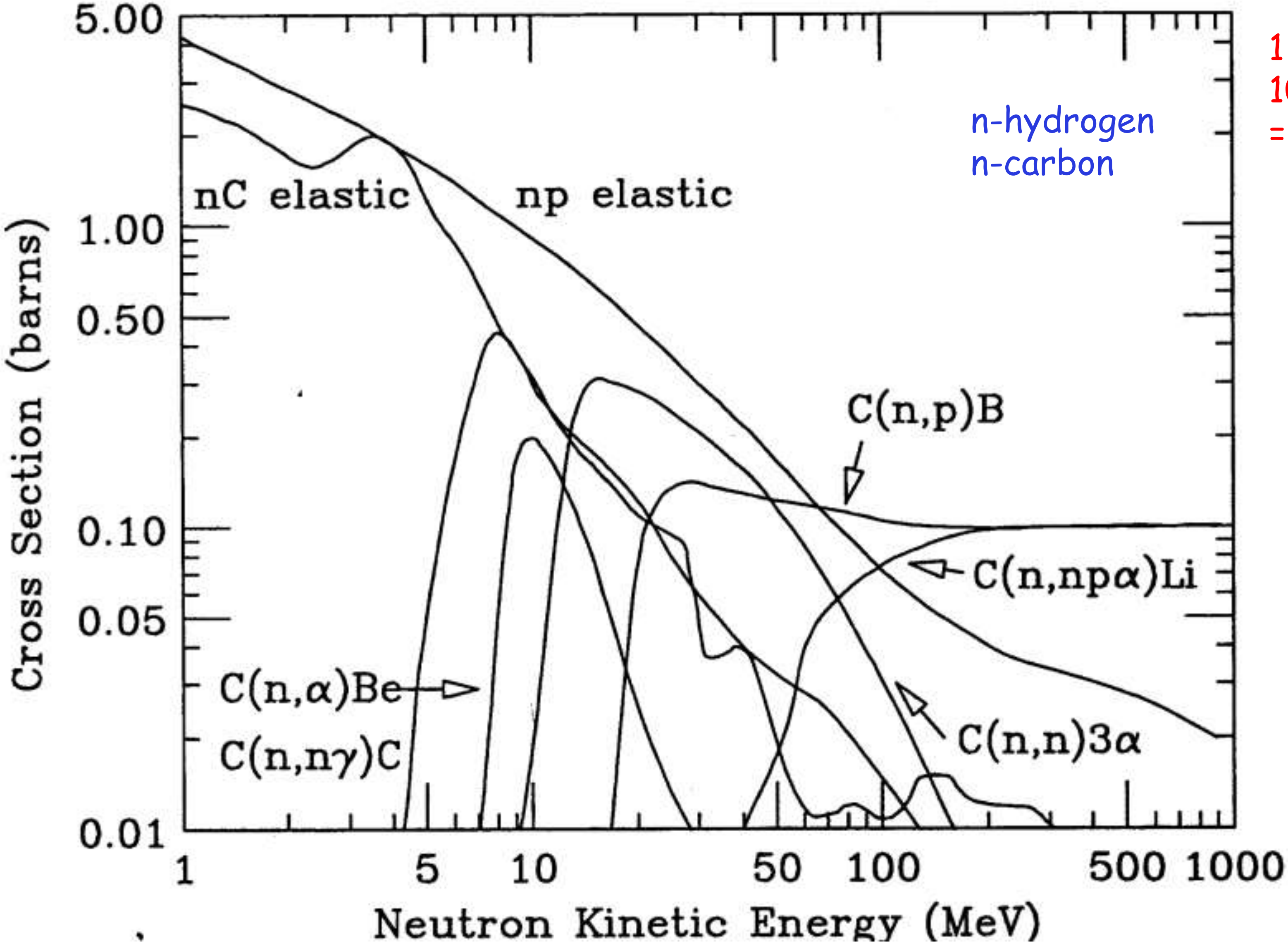
$E_n$ > 10 MeV Elastic+inel. scattering, various nuclear reactions, secondary charged reaction products

**Characteristic secondary nuclear radiation/products**:

1. $\gamma$-rays (n, $\gamma$)
2. charged particles (n,p), (n, $\alpha$),...
3. neutrons (n,n'), (n,2n'),...
4. fission fragments (n,f)



# Neutron Cross Sections

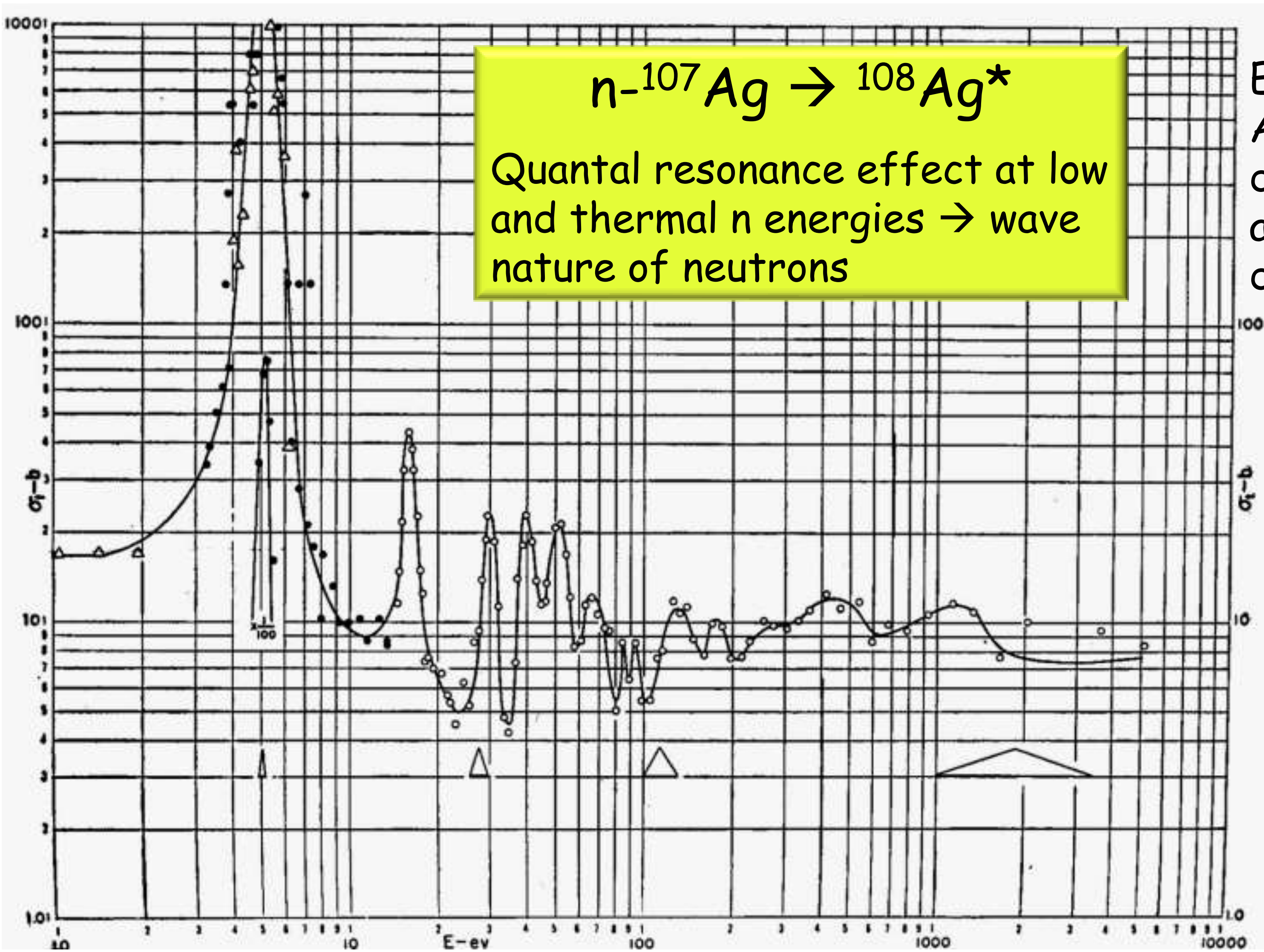



1 *b* = $10^{-24}cm^2$ $=100fm^2$

# Neutron Resonance Capture Cross Section

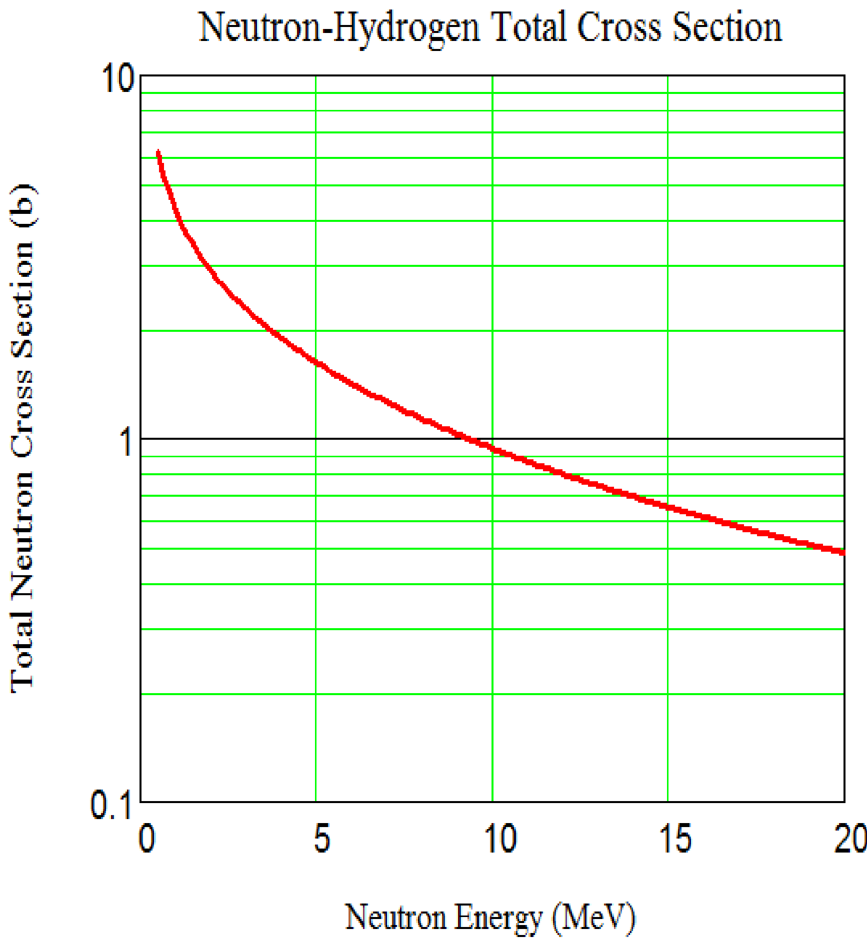
$n\text{-}^{107}Ag \rightarrow {}^{108}Ag^*$

Quantal resonance effect at low and thermal n energies → wave nature of neutrons




Excited Ag* nucleus deexcites and/or β-decays

# Total n-H Cross Section Parameterization



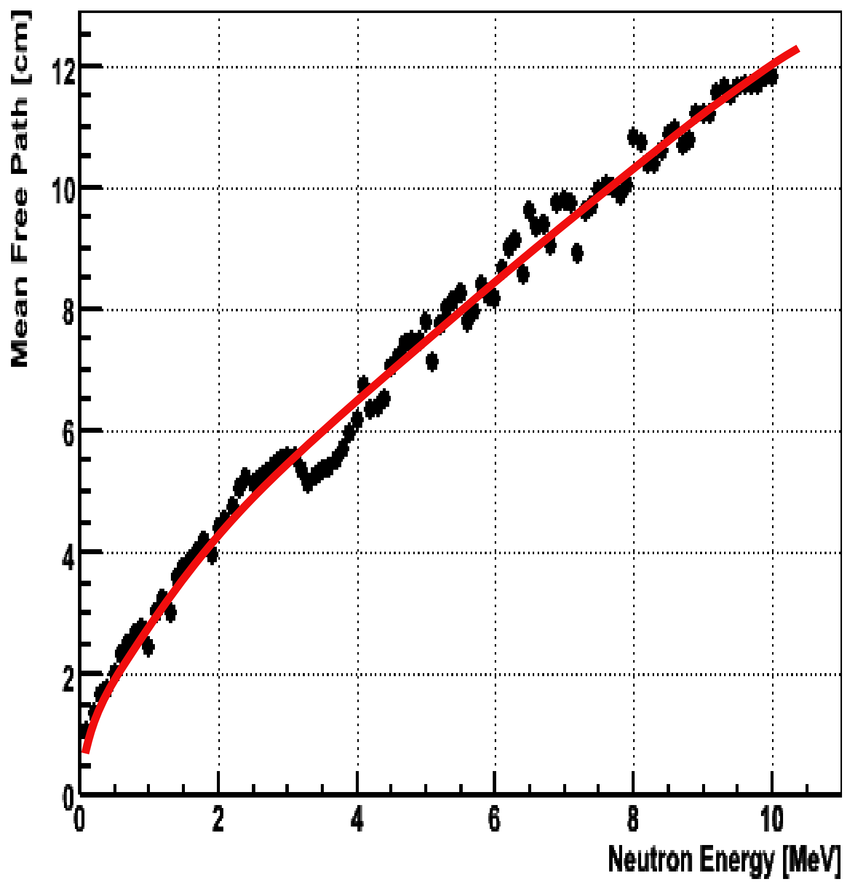

*Approximate parameterization for applications (detector efficiency estimates)*

*Cross section in barns (b)*
1b = $10^{-24}$cm$^2$

$$\sigma_n(E) = \frac{3\pi b}{1.206E + \left(-1.86 - 0.0941E + 0.0001306E^2\right)^2} + \frac{\pi b}{1.206E + \left(0.4223 + 0.13E\right)^2}$$

# Neutron Mean Free Path

## Mean Free Path of Neutrons in Water




$$\text{Survive w/o collision:}$$
$$\langle N(x)\rangle = N(0)\,e^{-x/\lambda}$$
$$\text{Gaussian Distribution}$$
$$\langle x\rangle = \lambda; \quad \sigma_x^2 \approx 2\cdot\lambda^2$$
$$\Gamma_{FWHM} = 2.35\cdot\sigma_x$$
$$\lambda = \frac{1}{\mu} = \frac{1}{\rho\sigma} \quad (mfp)$$

$\lambda$ = average path length in medium between 2 collisions

$\rho$: number density (atoms/volume)
$\sigma$: cross section

# Neutron Diffusion

Multiple scattering = statistical process

Heavy materials (A>>1): random scattering

$$N = \frac{1}{\xi} \ln\left(\frac{E_0}{E_1}\right)$$

Number of collisions $E_0 \rightarrow E_1$

Probability for no collision along path length x: P(x)

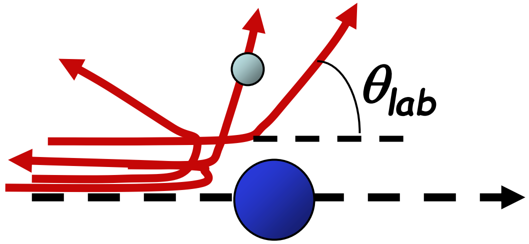
$$P(x) = \frac{1}{\lambda} \cdot e^{-\frac{x}{\lambda}}$$

for λ = const.

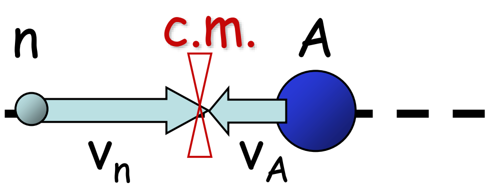
$$Mean - square\ displacement$$

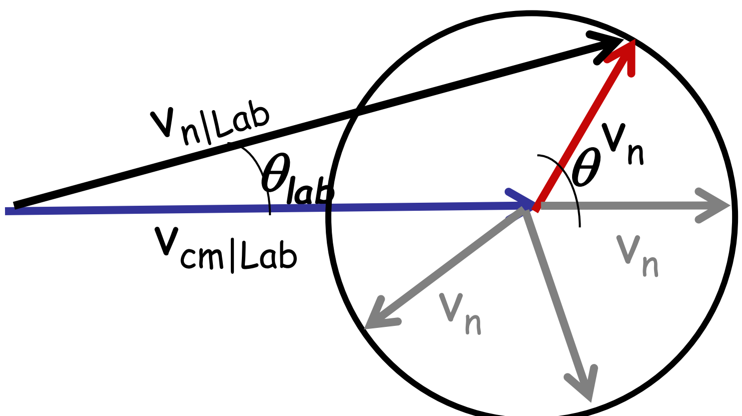
$$\left\langle x^2 \right\rangle_N = N \left\langle \lambda^2 \right\rangle = \int_0^\infty dx\, x^2 e^{-x/\lambda} \Big/ \int_0^\infty dx\, e^{-x/\lambda} = 2\lambda^2$$

# Energy Transfer in Elastic Scattering



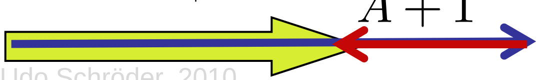

Neutron with lab velocity $v$, energy $E$, scatters randomly off target nucleus of mass number $A$ at rest in lab.




$$\max : v_{n|Lab} = v$$

$$\min \ : v_{n|Lab} = v \frac{A-1}{A+1}$$



$$c.m.: \ p_n = -p_A \qquad p_n + p_A = 0$$

$$v_n = v \frac{A}{A+1} \qquad v_A = -v \frac{1}{A+1}$$

$$v_{cm|Lab} = v \frac{1}{A+1}$$

lab velocity of center of gravity

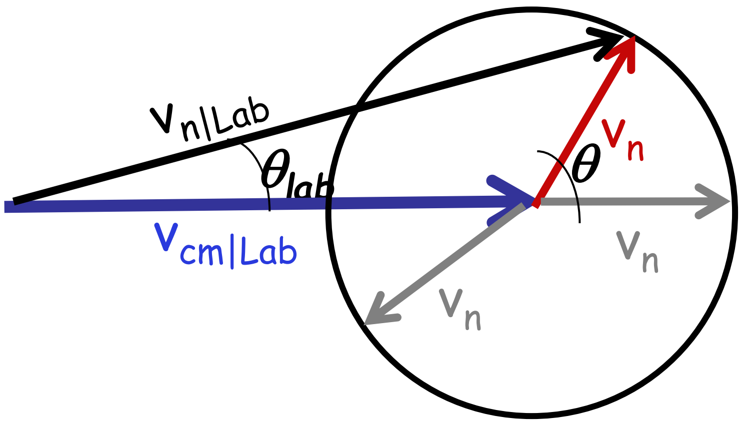
$$\max, \min : v_{n|Lab} = v_{cm|Lab} \pm v_n$$

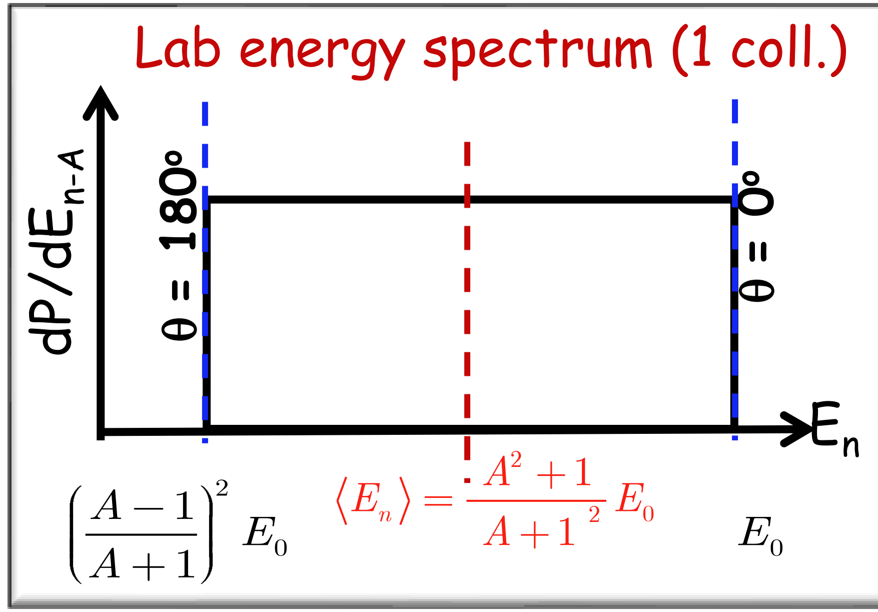
$$\max : \ E_{n|Lab} = E$$

$$\min \ : \ E_{n|Lab} = E \frac{\left(A-1\right)^2}{\left(A+1\right)^2}$$

# Scattered-Neutron Energy Spectrum

Neutron with energy $E_0$ scatters off target nucleus A at rest in lab.




$$E_{n|Lab} \propto v^2_{n|Lab} = \left(\vec{v} + \vec{v}_{cm|Lab}\right)^2 = \vec{v}^2 + \vec{v}^2_{cm|Lab} + 2\vec{v}\cdot\vec{v}_{cm|Lab}$$

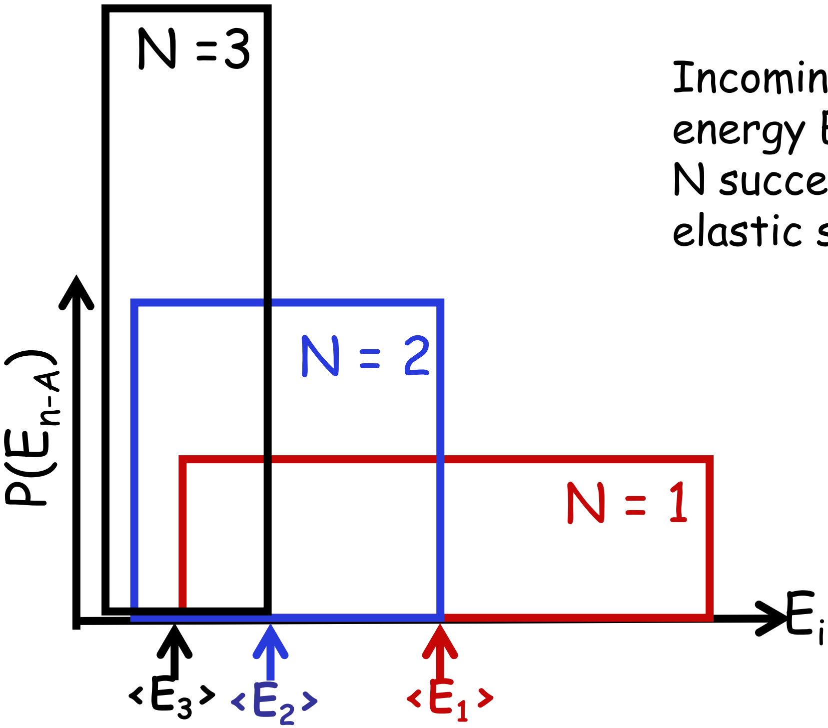
$$= \vec{v}^2 + \vec{v}^2_{cm|Lab} + 2v^2 \frac{A}{(A+1)^2}\cos\theta$$

$$\frac{dE_{n|Lab}}{d\theta} \propto \sin\theta \rightarrow \frac{dE_{n|Lab}}{d\Omega} \propto \frac{dE_{n|Lab}}{\sin\theta\, d\theta} = const.$$

$$\Rightarrow \frac{d\sigma_{n-A}(\theta)}{d\Omega} \propto \frac{d\sigma_{n-A}(E_{n|Lab})}{dE_{n|Lab}} \propto \frac{dP_{n-A}(E_{n|Lab})}{dE_{n|Lab}}$$

The *laboratory* energy spectrum of scattered n reflects the *center-of-mass* scattering angular distribution !

# Multiple n-A Scattering




Incoming neutron energy $E_0$
N successive elastic scatterings

$$\langle E_1 \rangle = \left[ \frac{A^2+1}{A+1\ ^2} \right]^1 E_0 = \alpha^1 E_0$$

$$\langle E_2 \rangle = \alpha \langle E_1 \rangle = \alpha^2 E_0$$

.................

$$\langle E_N \rangle = \alpha \langle E_{N-1} \rangle = \alpha^N E_0$$

$$\Rightarrow \boxed{P(E_1) = \frac{1}{1-\alpha\ \ E_0}}$$

$$\langle \ln E_N \rangle = \ln E_0 - N\xi$$

$$\rightarrow E_N \approx E_0 \cdot e^{-N\xi}$$

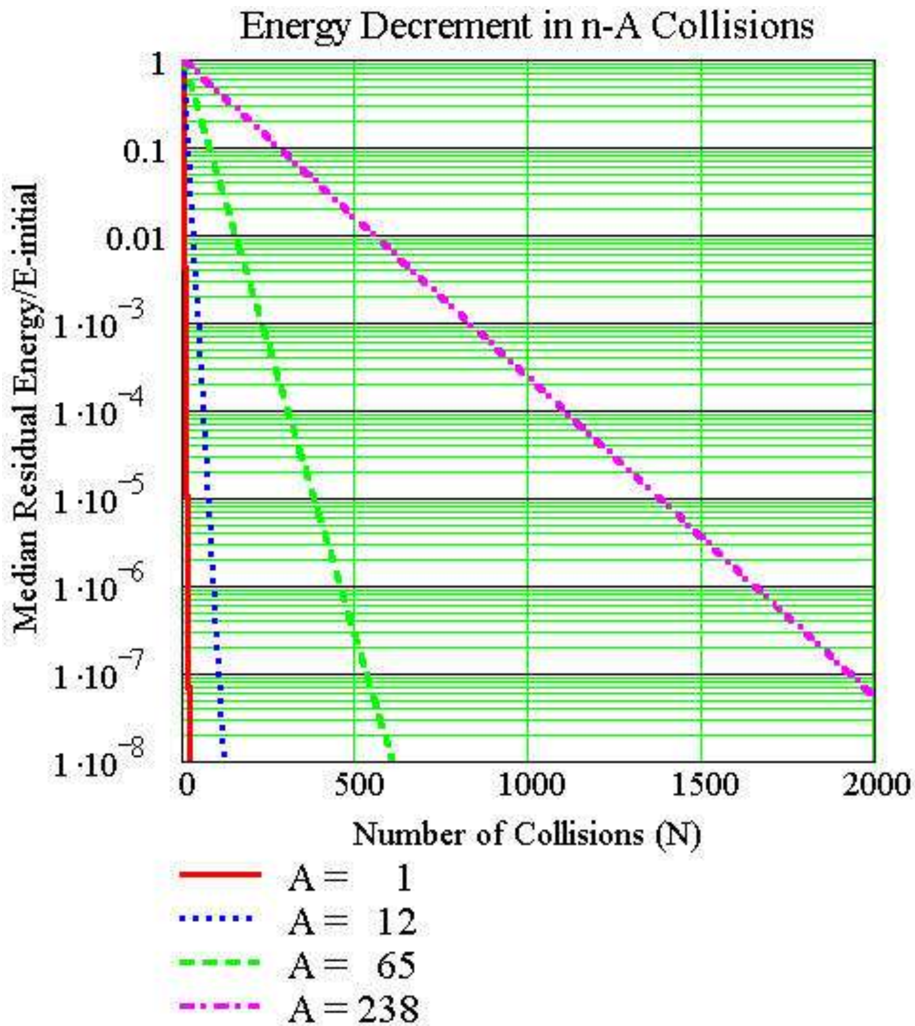
$$\xi := \left\langle \ln\ E_0 / E_1 \right\rangle = \int_{\alpha E_0}^{E_0} dE \ln\left(\frac{E_0}{E}\right) P\ \ E$$

$$= \frac{1}{1-\alpha} \int_1^{\alpha} dE \ln E = \frac{\left[x \ln x - x\right]_1^{\alpha}}{1-\alpha}$$

$$= 1 + \frac{\alpha \ln \alpha}{1-\alpha} = 1 + \frac{A^2+1}{2A} \ln\left(\frac{A^2+1}{(A+1)^2}\right) \xrightarrow[A>10]{} \frac{2}{A+2/3} \neq f(E_0)$$

"Logarithmic Decrement" ξ

# Thermalization Through Scattering




$$\left\langle \ln E_N \right\rangle = \ln E_0 - N\xi$$

$$Define\ \tilde{E}\ as\ median\,(<mean)$$

$$\ln \tilde{E} := \left\langle \ln E_N \right\rangle = \left\langle \ln E_0 \right\rangle - N\xi$$

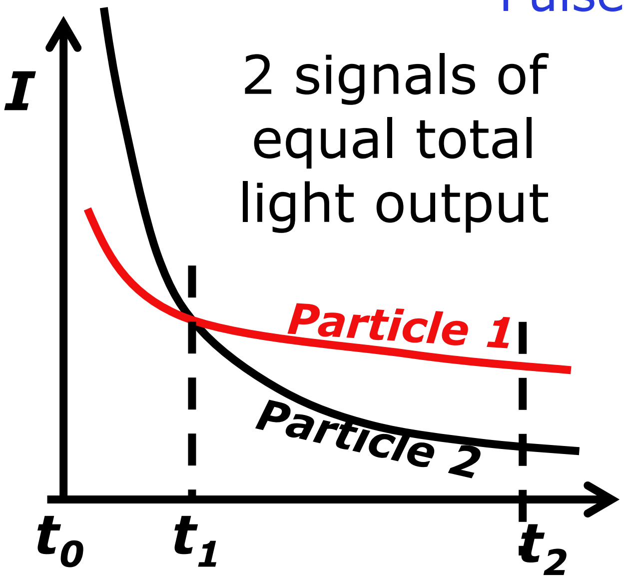
$$\boxed{\tilde{E}(N) = E_N = E_0 \cdot e^{-N\xi}}$$

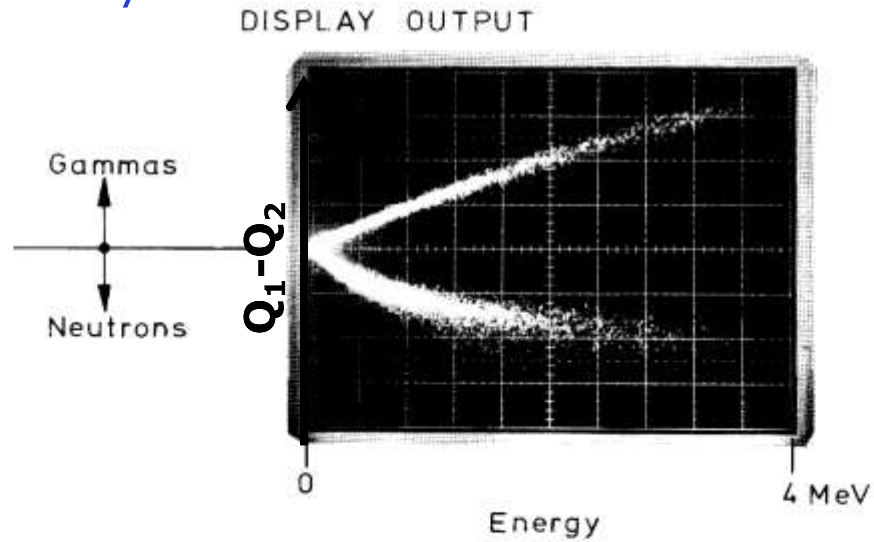
N-therm: $E_0$=2 MeV → 0.025eV

| A | ξ | N-therm |
|---|---|---|
| 1 | 1.0000 | 18 |
| 12 | 0.1578 | 115 |
| 65 | 0.0305 | 597 |
| 238 | 0.0084 | 2172 |

# n-Identification in Scintillators (PSD)

## Pulse Shape Analysis



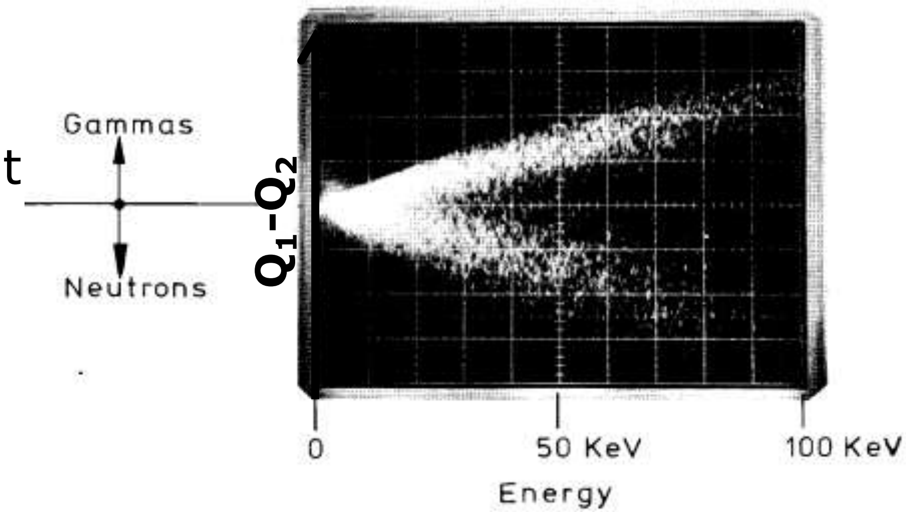




$$Q_1 = \int_{t_0}^{t_1} I(t)dt$$

fast component

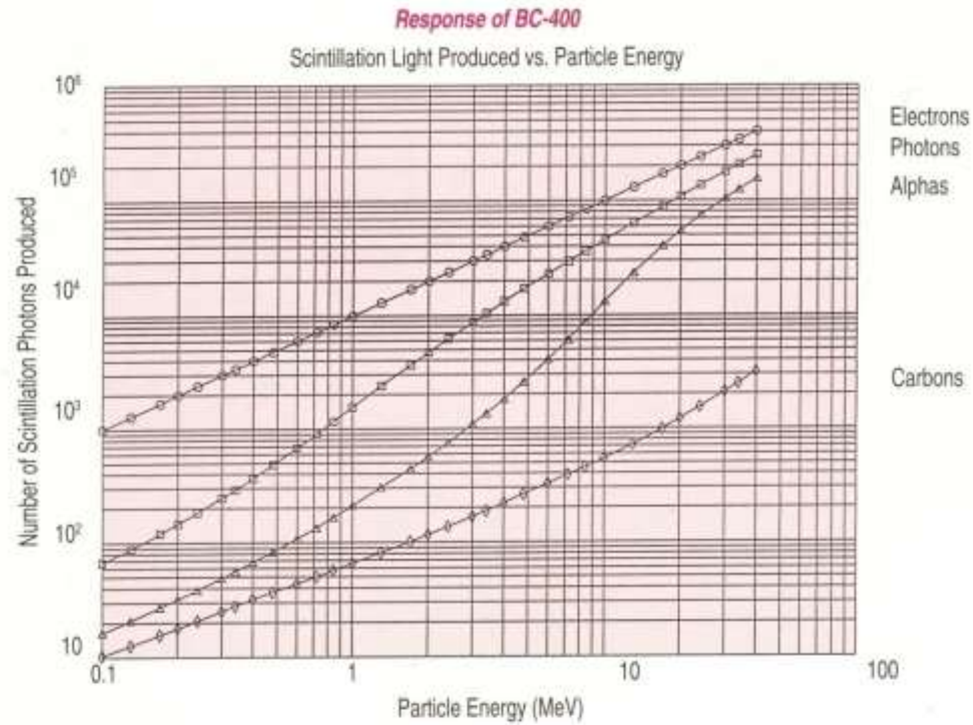
slow

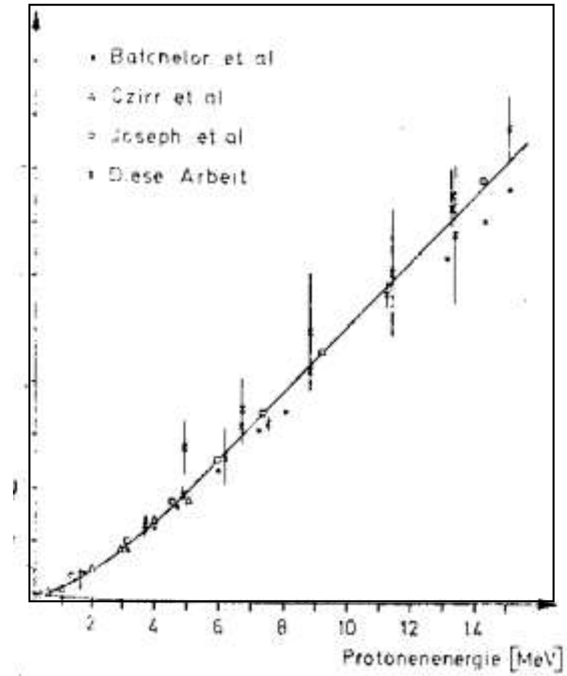
$$Q_2 = \int_{t_1}^{t_2} I(t)dt$$




$$Q_1 + Q_2 = Q \propto L(Energy)$$

# Scintillator LO Response to $\gamma$-Rays and Particles



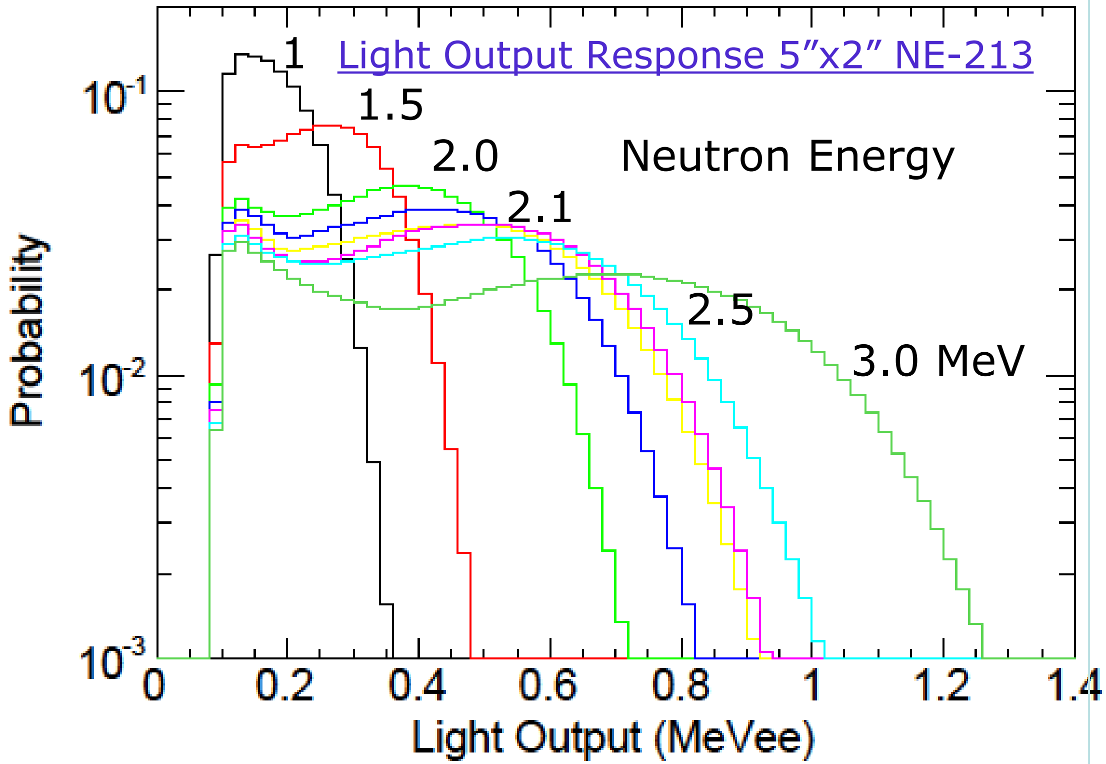

For a given energy, electrons & photons have the highest light output

Non-linear response to massive particles

NE 213 liquid scintillator:
electron-equivalent energies $E_e \leftrightarrow E_p$

$$E_e(E_p) = \begin{cases} \left(0.18MeV^{-1/2}\right) E_p^{3/2} & E_p < 5.25MeV \\ 0.63E_p - 1.10MeV & E_p \geq 5.25MeV \end{cases}$$

# Thin-Detector Light Output Response




Equivalent to full-energy peak?
→ Thick detector (many $\lambda$ thick)

**Thin detector:**

**1 n-p interaction within detector** (n leaves)

Equivalent to $\gamma$-e Compton

Each fixed neutron energy produces recoil proton energy ($E_p$) distribution → produces distribution in LO (light output).

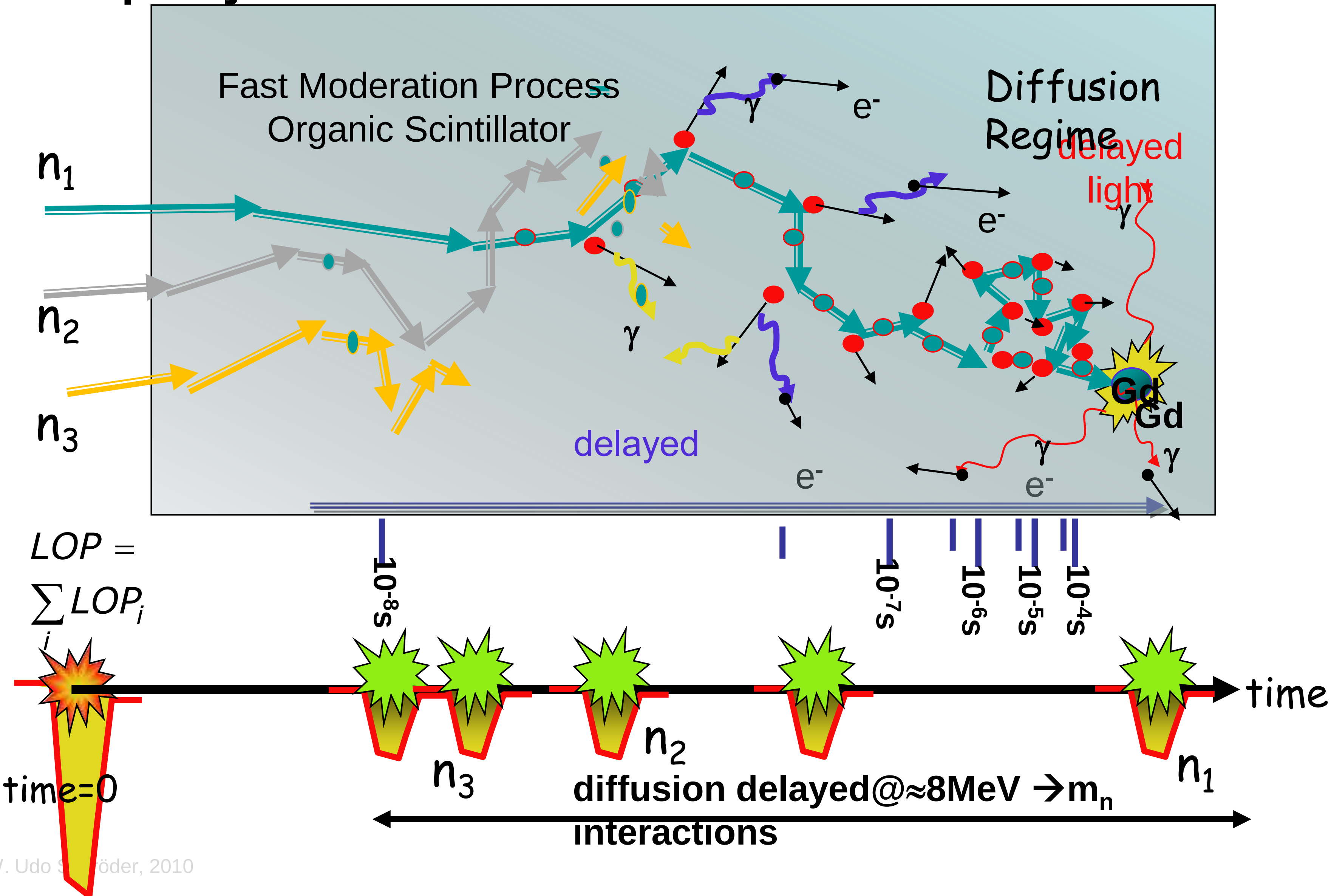
LO response is calibrated with $\gamma$-rays → electron-equivalent energy $E_{ee}$ ($eV_{ee}$).

Convert measured $E_{ee}$ → $E_p$

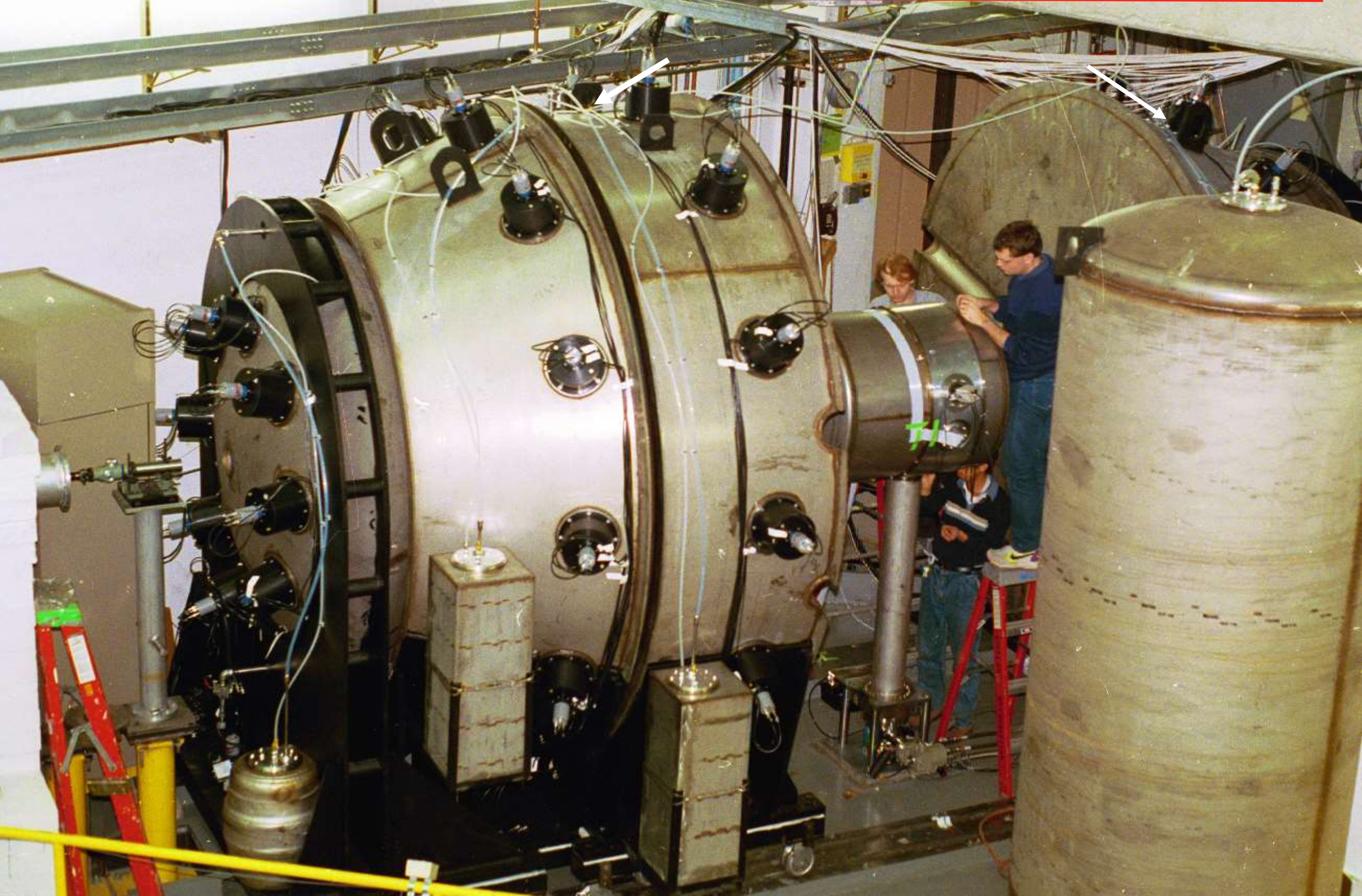
Unfold $E_p$ distribution

# n-Stopping and Scintillation Process in Thick Detector

**Prompt Injection of m neutrons**

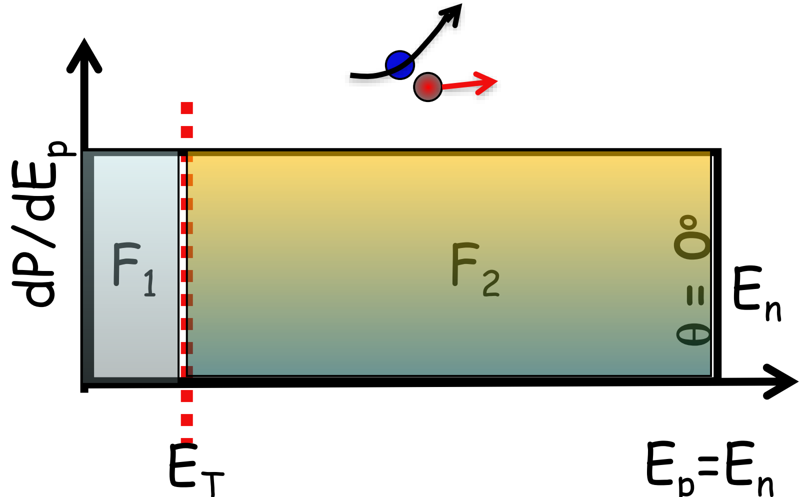


# SuperBall Neutron Calorimeter

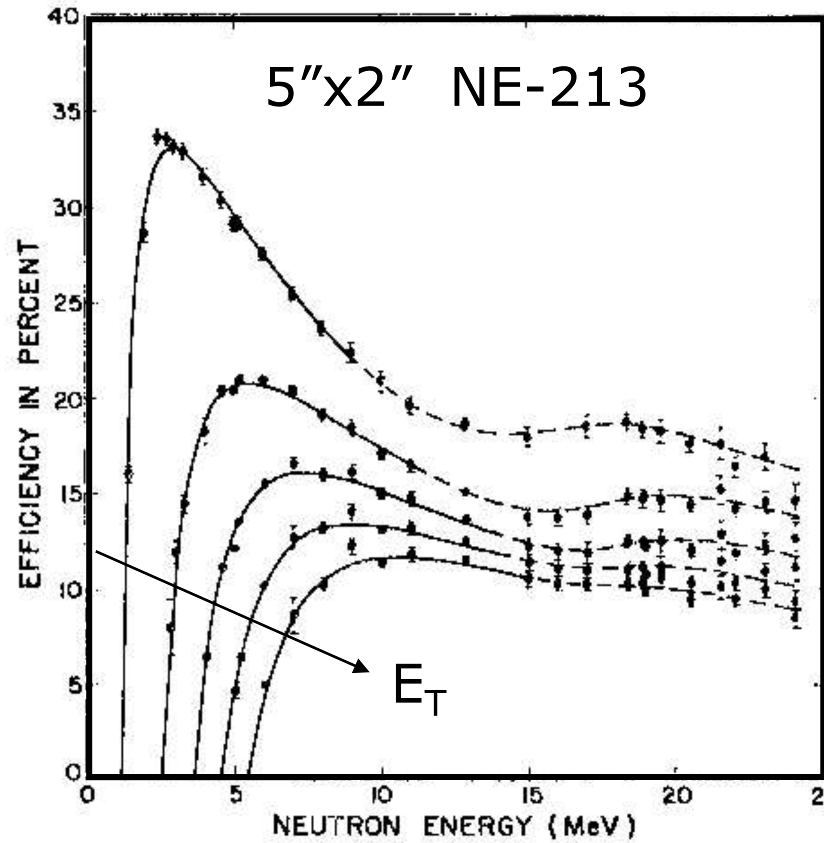


# Efficiency of *p*-Recoil NeutronDetectors

angle dependent n-p energy transfer → continuous recoil energy spectrum




Electronic detector threshold



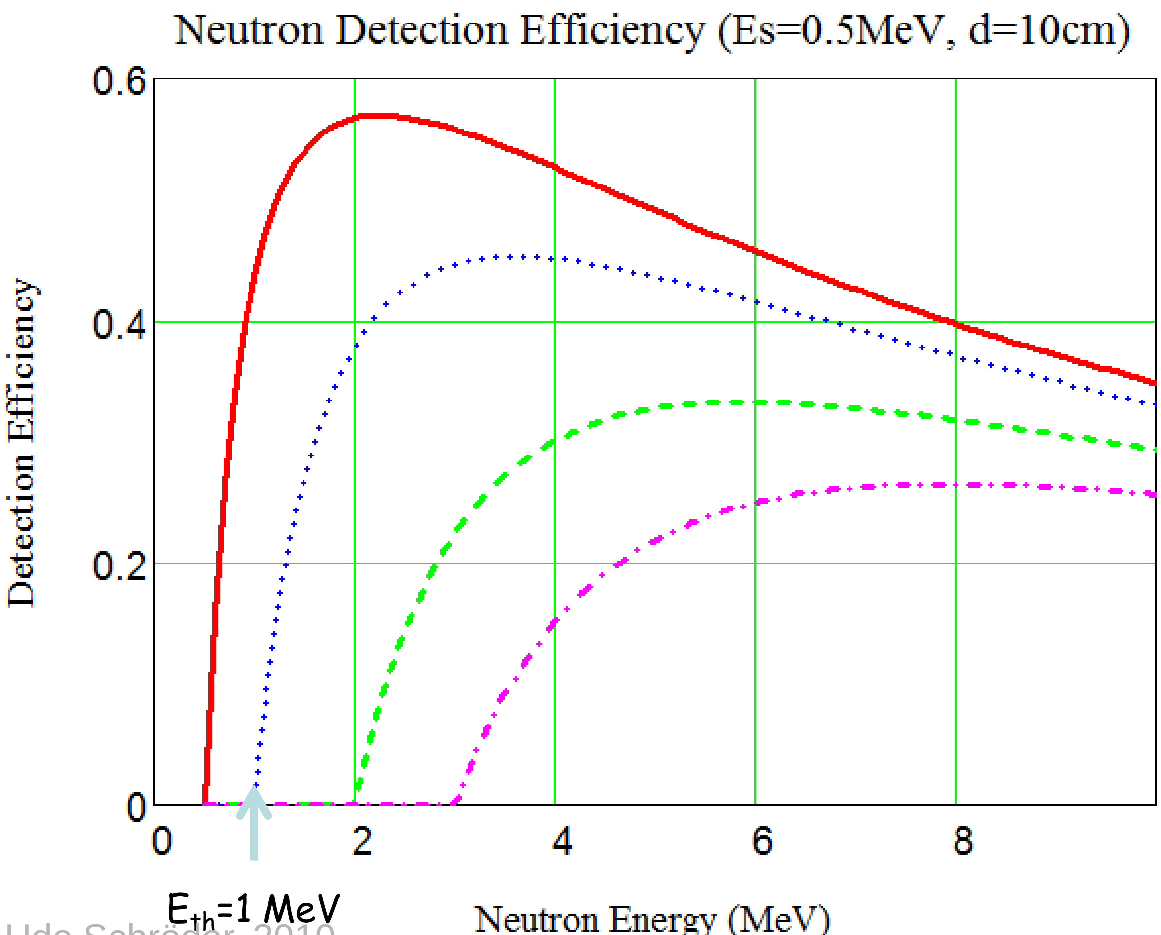

$$\varepsilon(E_n, E_T) = \frac{F_2}{F_1 + F_2}$$

$$\approx \sigma(E_n)\left[1 - \frac{E_T}{E_n}\right]$$

$$\sigma(E_n) = \sum_{X,y} \sigma_{X(n,y)}(E_n) \;\; all \;\; n - induced$$

# Detector Efficiency Estimates

$$\varepsilon(E, E_{Th}, d) \approx \left(1 - \frac{E_{Th}}{E}\right) \cdot \{1 - \exp[-\sigma_n(E) \cdot n_H \cdot d]\}$$



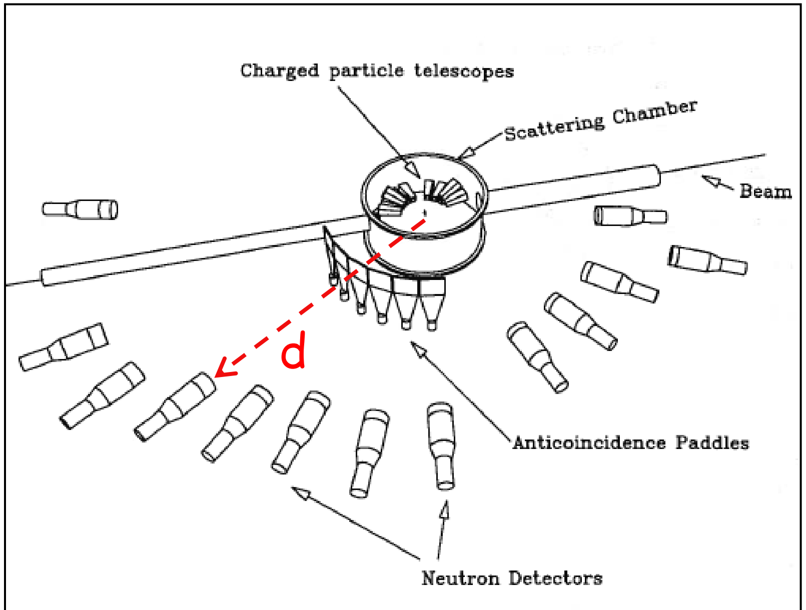

$E_{th}$=1 MeV

Detector thickness
d =10cm
Hydrogen density (atoms/cm$^3$) NE-213
$n_H = 4.86 \cdot 10^{22}/cm^3$

Approximate intrinsic detection efficiency ε(E)
Probability for detection if incident neutron trajectory is perpendicular to detector face.

Total efficiency contains ε(E) and solid-angle factor ΔΩ.

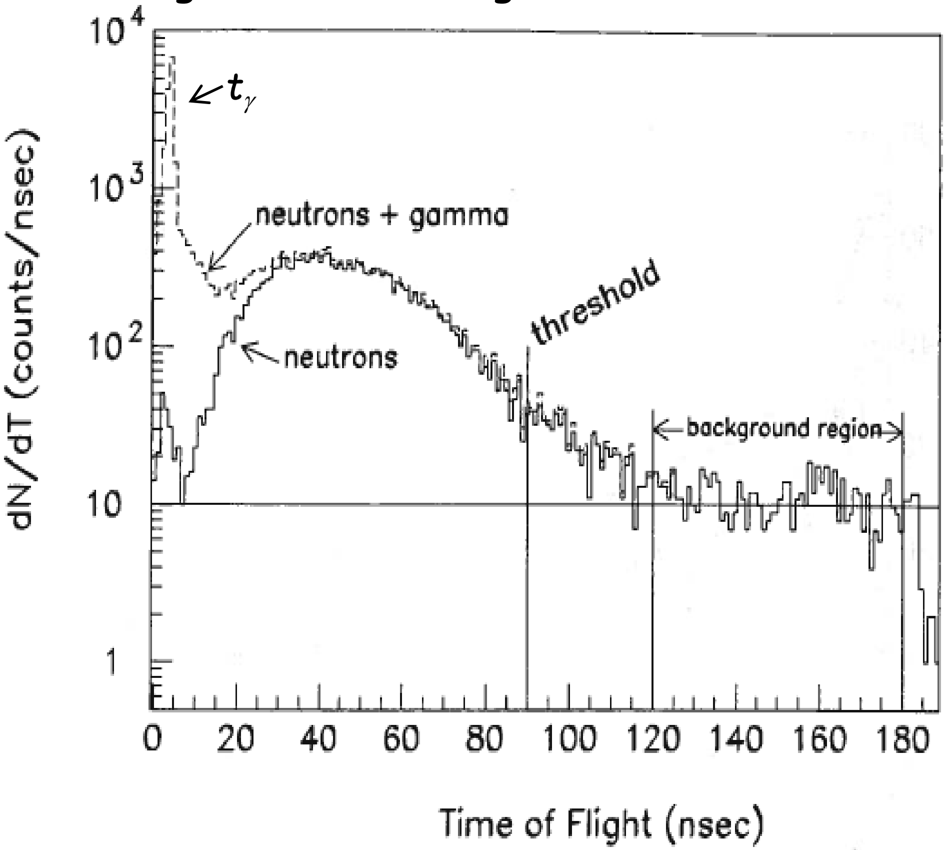
# Associated-Particle/Neutron TOF



Pictorial view of the experimental setup.

Reference time-zero $t_0$: true TOF = $t_{meas}-t_0$
a) from accelerator signal,
b) associated particle* of known E
c) $\gamma$-ray* (v=c)
*measured with same or different detector

d= target-detector flight distance




$$Non-relativistically:$$

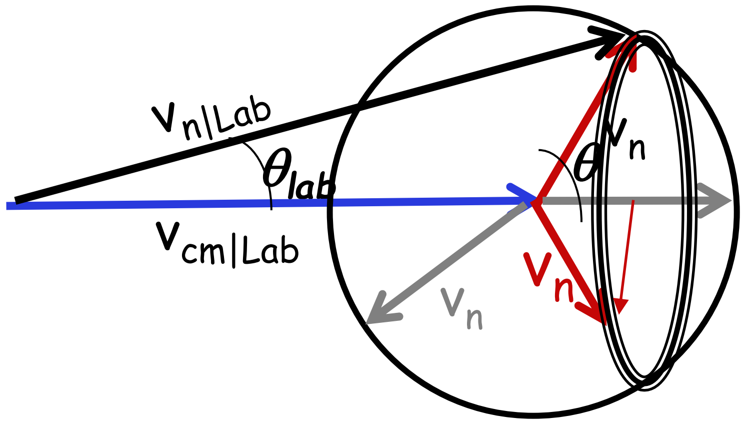
$$E = \frac{m}{2}\upsilon^2 = \frac{m}{2}\left(\frac{d}{t-t_0}\right)^2$$

$$Spectrum$$

$$\frac{dN(E)}{dE} = \frac{dN\left(t(E)\right)}{dt}\cdot\frac{dt}{dE}$$

# Applications

# n Angular Distribution




$$v_n = \frac{v}{A+1} A \qquad v_{cm|Lab} = \frac{v}{A+1}$$

$$\vec{v}^{\,2}_{n|Lab} = \left(\vec{v}_n + \vec{v}_{cm|Lab}\right)^2 = $$

$$= \frac{v^2}{(A+1)^2}\left[A^2 + 1 + 2A\cos\theta\right]$$

$$v_n^2 = v_{n|Lab}^2 + v_{cm|Lab}^2 - 2v_{n|Lab}v_{cm|Lab}\cos\theta_{Lab} \qquad |: v^2 / (A+1)^2$$

$$A^2 = \left[A^2 + 1 + 2A\cos\theta\right] + 1 - 2\sqrt{A^2 + 1 + 2A\cos\theta}\,\cos\theta_{Lab}$$

$$\cos\theta_{Lab} = \frac{1 + A\cos\theta}{\sqrt{A^2 + 1 + 2A\cos\theta}}$$

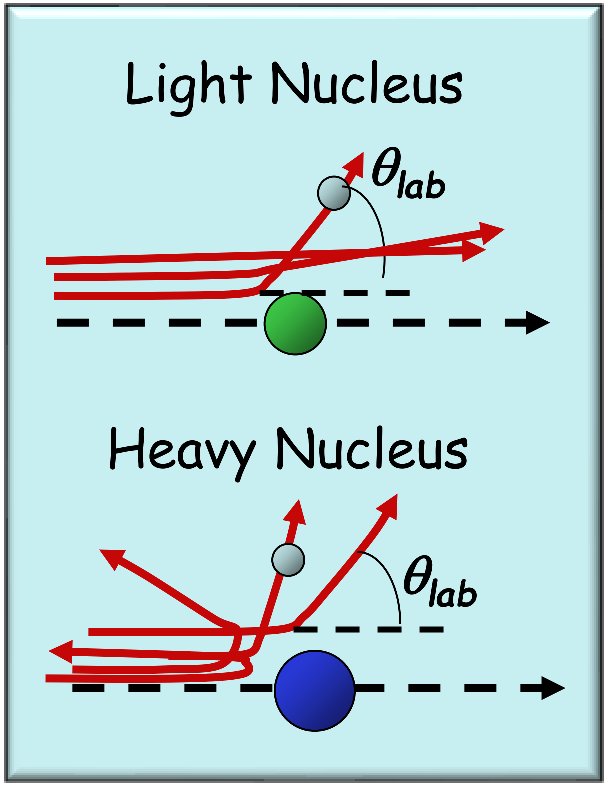
$$\langle\cos\theta_{Lab}\rangle = \frac{1}{2}\int_{-1}^{+1} d\cos\theta\, \frac{1 + A\cos\theta}{\sqrt{A^2 + 1 + 2A\cos\theta}} = \frac{2}{3A}$$

> 0 → forward scattering

# A Dependence of Angular Distribution

Properties of n scattering depends on the sample mass number *A*

➔ Measure time-correlated flux of transmitted or reflected neutrons




*Average scattering* ∡

$$\langle \cos\theta_{lab} \rangle = 2/(3A) \propto A^{-1}$$

$$\langle \theta_{lab} \rangle \sim \frac{\pi}{2} - \frac{2}{3A}$$

$$\tilde{E}(N) \approx E_0 \cdot \exp\left\{\frac{-2N}{(A+2/3)}\right\}$$

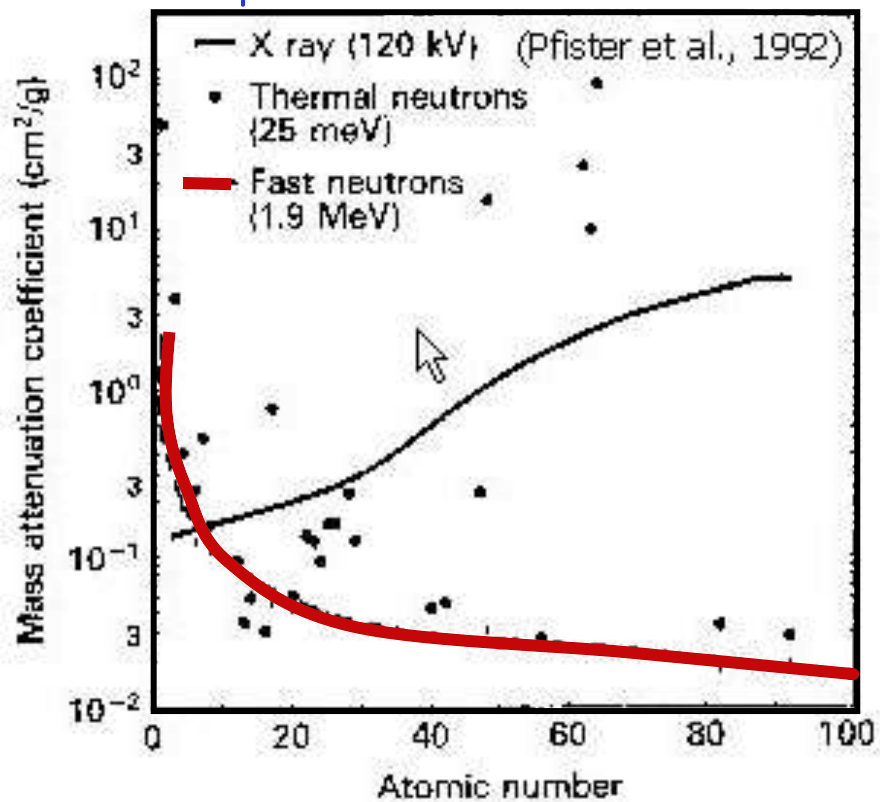
(*After N collisions*)

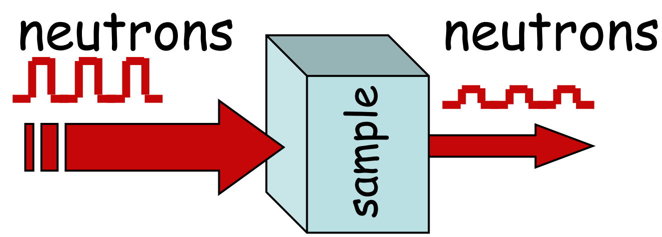
Light nuclei: slowing-down and diffusion of neutron flux

Heavy nuclei: neutrons lose less energy, high reflection & transmission

# Principle of Fast-Neutron Radiography(Imaging)

Comparison of different radiations




$\phi(0)$ → $\delta$ ← $\phi(\delta)$

incoming transmitted

$$\phi(\delta) = \phi(0) \cdot T(\delta)$$

$$T(\delta) = e^{-\Sigma \cdot \delta} \quad Transmission$$

$$\Sigma = \mu \cdot \rho = N_A \cdot \sigma_A \cdot \rho$$

$$\mu = atten.coeff$$

$$\rho = material\ density$$

**Transmission decreases exponentially (reflectivity increases) with thickness and density of sample.**
**Neutrons more penetrating → use for thick samples**

# Commercial Neutron Generator ING-03

Neutrons can be produced in a variety of reactions, e.g., in nuclear fission reactors or by the D(d,n)$^3$He or T(d,n)$^4$He reactions




Specs: < $3 \cdot 10^{10}$ D(d,n)$^3$He neutrons/s Total yield $2 \cdot 10^{16}$ neutrons

Pulse frequency 1-100Hz Pulse width > 0.8 μs, Power 500 W

Alternative option:
T(d,n)$^4$He, $E_n \approx 14.5$ MeV

Source: All-Russian Research Institute of Automatics **VNIIA**